MATHEMATICS-IX

Module - 7

INDEX

Shrivastava Classes, D-27, Near JVTS Garden,Chattarpur Extension
New Delhi - 110074
9810934436 , 8076575278 , 8700391727

HERON'S FORMULA

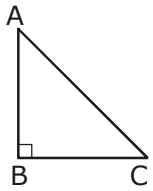
INTRODUCTION

In earlier classes, we have learnt to find perimeter and area of various plane figures such as triangles, quadrilaterals-rectangle, square, parallelogram, trapezium etc. But we know to find the area of a triangle only when its altitude is given or the triangle is right angled, isosceles or equilateral.

In the present chapter, we shall study to find the area of any kind of triangle. Then with the help of area of triangle we will find the area of different plane figures which can be reduced to different triangles or quadrilaterals.

AREA OF SOME SPECIFIC PLANE FIGURES :

1. Right Angled Triangle



Area of a right angled triangle

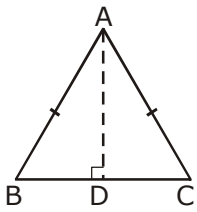
$= \frac{1}{2} \times$ base $\times$ altitude

$= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2}$ (product of sides forming right angle).

2. Isosceles Triangle :

Let ABC be an isosceles triangle with AB = AC. Let AD $\perp$ BC. Then, by simple geometry, we can prove that AD bisects BC. Then BD = $\frac{1}{2}$BC. Let a be the equal side and b be the base then BD = $\frac{1}{2}$b and AB = a.



$\therefore$ By Pythagoras theorem in $\triangle$ABD, we have

$AB^2 = AD^2 + BD^2$

$\Rightarrow \quad a^2 = AD^2 + \frac{b^2}{4} \quad \Rightarrow \quad AD = \sqrt{a^2 - \frac{b^2}{4}}$

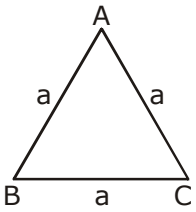
$\therefore$ area of isosceles $\triangle$ABC

$= \frac{1}{2} \times BC \times AD \quad = \frac{1}{2} \times b \times \sqrt{a^2 - \frac{b^2}{4}}$

$= \frac{1}{2} \times$ base $\times \sqrt{(\text{equal side})^2 - \frac{(\text{base})^2}{4}}$

3. Equilateral Triangle :

Let a be the side of an equilateral triangle. Then putting b = a in area for isosceles triangle, we get area of equilateral triangle

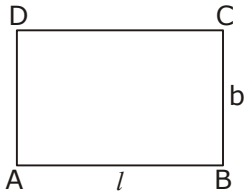


$$= \frac{1}{2} \times a \times \sqrt{a^2 - \frac{a^2}{4}} \qquad = \frac{1}{2} a \frac{\sqrt{3}}{2} \cdot a \qquad = \frac{\sqrt{3}}{4} a^2$$

$\therefore$ area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Rectangle :

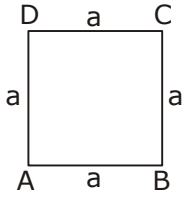
If l and b be the length and breadth of a rectangle, then



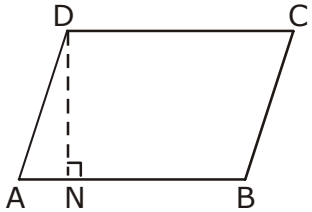
area of rectangle = $l \times b$.

5. Square :

If a be the side of a square, then area of square = $(\text{side})^2 = a^2$.



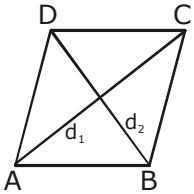
6. Parallelogram:



Area of a parallelogram = base × corresponding altitude = AB × DN.

7. Rhombus :

In d_1 and d_2 are the diagonals of a rhombus then area of a rhombus



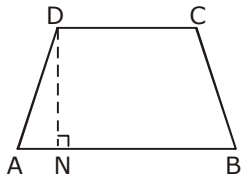
$$= \frac{1}{2}(d_1 \times d_2)$$

$$= \frac{1}{2}(AC \times BD)$$

(Note : For rhombus we can use the formula as we have for parallelogram, as rhombus is also a parallelogram, provided the measures of the base and the corresponding altitude are known.)

8. Trapezium :

If in trap. ABCD, AB || CD and DN is the distance between parallel sides then area of trapezium

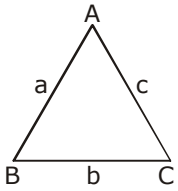


$= \frac{1}{2} \times$ sum of parallel side $\times$ distance between them.

$= \frac{1}{2} \times (AB + DC) \times DN.$

9. Heron's Formula

Heron, a mathematician, gave the famous formula for finding the area of any kind of triangle is terms of its three sides. After his name the formula is known as Heron's formula which is discussed below.



If a, b, c are the sides of a triangle then area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where s is the semiperimeter of triangle that is

$$s = \frac{a+b+c}{2}$$

(Note. In case we know all sides of a triangle as well as altitude corresponding to a particular side, it is better to use the formula.

area of triangle = $\frac{1}{2} \times$ base $\times$ altitude instead of using Heron's formula as it saves a lot of calculations.)

Applicability of Heron's formula will be clear through the following examples.

HERON'S FORMULA

If a,b,c denote the lengths of the sides of a triangle ABC. Then,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$ is the semi–perimeter of ΔABC.

Note: This formula is applicable to all types of triangles whether it is scalene or equilateral or isosceles.

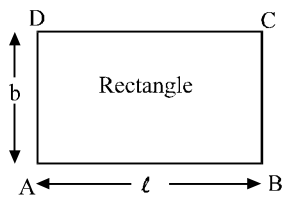
RECTANGLE

If ℓ and b denote respectively the length and breadth of a rectangle, then

(i) Perimeter = $2(\ell + b)$

(ii) Area = $\ell \times b$

(iii) $(\text{Diagonal})^2 = (\text{Length})^2 + (\text{Breadth})^2$



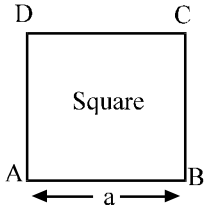
SQUARE

If "a" denote the length of each side of a square then,

(i) Perimeter = 4a

(ii) Area = a^2 = $(side)^2$

(iii) Area = $\frac{1}{2}$ $(diagonal)^2$



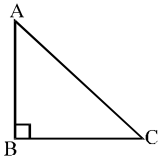
RIGHT-ANGLED TRIANGLE

Let ABC be a right triangle right angled at B. Then,

(i) Perimeter = AB + BC + CA

(ii) Area = $\frac{1}{2}$ (Base × Height)

= $\frac{1}{2}$ × (BC × AB)



ISOSCELES TRIANGLE

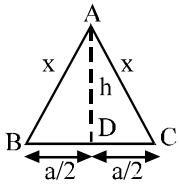
Let ABC be an isosceles triangle such that AB = AC = x and BC = a. Then,

(i) Perimeter = AB + BC + CA

= a + 2x

(ii) Area = $\frac{1}{2}$ (Base × Height)

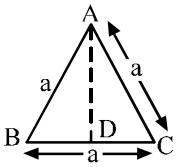
= $\frac{1}{2}$ × $\left[a \times \sqrt{x^2 - \frac{a^2}{4}}\right]$



(iii) h = $\sqrt{x^2 - (a/2)^2}$

EQUILATERAL TRIANGLE

Let ABC be an equilateral triangle of each side a. Then,



(i) Perimeter = 3a (ii) Altitude = $\frac{\sqrt{3}}{2}$ a (iii) Area = $\frac{\sqrt{3}}{4}$ a^2

SOLVED PROBLEMS

Ex.1 Find the area of a triangle whose sides are 13 cm, 14 cm and15 cm

Sol. Let a, b, c be the sides of the given triangle and s be its semi-perimeter such that

a = 13 cm, b = 14 cm and c = 15 cm

Now, $s = \frac{1}{2}(a + b + c) = \frac{1}{2}(13 + 14 + 15) = 21\text{cm}$

$\therefore$ s – a = 21 – 13 = 8cm, s – b = 21 – 14 = 7cm and s – c = 21 – 15 = 6cm

Hence, Area of given triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21\times8\times7\times6} = \sqrt{7\times3\times8\times7\times2\times3} = \sqrt{7^2\times4^2\times3^2}$

$= 7 \times 4 \times 3 = 84\ \text{cm}^2$

Ex.2 Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm.

Sol. Let a, b, c be the sides of the given triangle and 2s be its perimeter such that

a = 8 cm, b = 11 cm and 2s = 32 cm i.e. s = 16 cm

Now, a + b + c = 2s

$\Rightarrow$ 8 + 11 + c = 32

$\Rightarrow$ c = 13cm

$\therefore$ s – a = 16 – 8 = 8cm, s – b = 16 – 11 = 5cm and s – c = 16 – 13 = 3cm

Hence, Area of given triangle = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{16\times8\times5\times3} = \sqrt{8\times8\times30} = 8\sqrt{30}\ \text{cm}^2$

Ex.3 The perimeter of a triangular field is 450 m and its sides are in the ratio 13 : 12 : 5. Find the area of triangle.

Sol. It is given that the sides a,b,c of the triangle are in the ratio 13 : 12 : 5 i.e.,

a : b : c = 13 : 12 : 5 $\Rightarrow$ a = 13x, b = 12x and c = 5x

$\therefore$ Perimeter = 450 $\Rightarrow$ 13x + 12x + 5x = 450 $\Rightarrow$ 30x = 450 $\Rightarrow$ x = 15

So, the sides of the triangle are

a = 13 × 15 = 195 m, b = 12 × 15 = 180 m and c = 5 × 15 = 75 m

It is given that perimeter = 450 $\Rightarrow$ 2s = 450 $\Rightarrow$ s = 225 m

Hence, Area = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{225(225-195)(225-180)(225--75)}$

$\Rightarrow$ Area = $\sqrt{225\times30\times45\times150} = \sqrt{5^2\times3^2\times3\times5\times2\times3^2\times5\times5^2\times2\times3}$

$\Rightarrow$ Area = $\sqrt{5^6\times3^6\times2^2} = 5^3\times3^3\times2 = 6750\ \text{m}^2$

Ex.4 Find the area of a triangle having perimeter 32 cm, one side 11 cm and difference of other two sides is 5 cm.

Sol. Let a, b and c be the three sides of ΔABC.

a = 11 cm

a + b + c = 32 cm $\Rightarrow$ 11 + b + c = 32 cm or b + c = 21 cm ... (1)

Also, we are given that b – c = 5 cm ... (2)

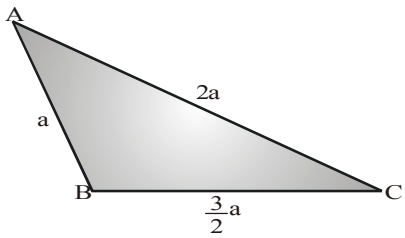
Adding (1) and (2), 2b = 26 cm i.e., b = 13 cm and c = 8 cm

Now, $s = \frac{a+b+c}{2} = \frac{11+13+8}{2} = \frac{32}{2} = 16$ cm

(s – a) = (16 – 11) cm = 5 cm, (s – b) = (16 – 13) cm = 3 cm and (s – c) = (16 – 8) cm = 8 cm

$\therefore$ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{16\times5\times3\times8}\ \text{cm}^2 = \sqrt{64\times30}\ \text{cm}^2 = 8\sqrt{30}\ \text{cm}^2$

Ex.5 In figure, find the area of the ΔABC.



Sol. $s = \frac{BC + CA + AB}{2} = \frac{\frac{3}{2}a + 2a + a}{2} = \frac{9}{4}a$

$\{s - (BC)\} = \frac{9}{4}a - \frac{3}{2}a = \frac{3}{4}a$

$\{s - (CA)\} = \frac{9}{4}a - 2a = \frac{1}{4}a$

$\{s - (AB)\} = \frac{9}{4}a - a = \frac{5}{4}a$

Now, area of $\Delta ABC = \sqrt{\frac{9}{4}a \times \frac{3}{4}a \times \frac{1}{4}a \times \frac{5}{4}a} = \sqrt{\frac{9 \times 3 \times 5}{4 \times 4 \times 4 \times 4}a^4} = \frac{3\sqrt{15}}{4 \times 4}a^2$ sq. units. $= \frac{3\sqrt{15}}{16}a^2$ sq. units.

Ex.6 The sides of a triangle are in the ratio 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Sol. Let us take the sides of the triangle as 3x, 5x and 7x because the ratio of the sides is given to be 3 : 5 : 7. Also, we are given that

$3x + 5x + 7x = 300 \Rightarrow 15x = 300 \Rightarrow x = 20$

Hence, the lengths of the three sides are 3 × 20 m, 5 × 20 m, 7 × 20 m. i.e., 60 m, 100 m, 140 m.

Now, $s = \frac{60 + 100 + 140}{2}$ m = 150 m

Area of the triangle

$= \sqrt{50 \times (150 - 60) \times (150 - 100) \times (150 - 140)}$ m^2

$= \sqrt{50 \times 90 \times 50 \times 10}$ m^2 $= \sqrt{15 \times 9 \times 5 \times 10000}$ m^2

$= 15 \times 100 \times \sqrt{3}$ m^2 $= 1500\sqrt{3}$ m^2.

Ex.7 The lengths of the sides of a triangle are 5 cm, 12 cm and 13 cm. Find the length of perpendicular from the opposite vertex to the side whose length is 13 cm.

Sol. Here, a = 5, b = 12 and c = 13.

$\therefore \quad s = \frac{1}{2}(a + b + c) = \frac{1}{2}(5 + 12 + 13) = \frac{30}{2} = 15$ cm

Let A be the area of the given triangle. Then,

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(15-5)(15-12)(15-13)}$

$\Rightarrow \quad A = \sqrt{15 \times 10 \times 3 \times 2} = 30$ cm^2 ... (i)

Let p be the length of the perpendicular from vertex A on the side BC. Then,

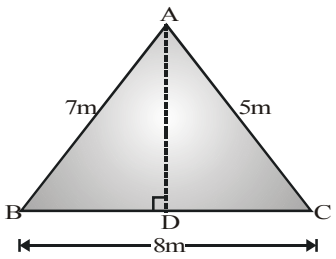
$A = \frac{1}{2} \times (13) \times p$ ... (ii)

From (i) and (ii), we get $= \frac{1}{2} \times 13 \times p = 30 \Rightarrow p = \frac{60}{13}$ cm.

Ex.8 In figure, there is a triangular childern park with sides, AB = 7 m, BC = 8 and AC = 5m, AD ⊥ BC and AD meets BC at D. Trees are planted at A, B, C and D. Find the distance between the trees at A and D.

Sol. In figure, a = 8 m, b = 5 m and c = 7 m.

$s = \frac{8+5+7}{2}\text{ m} = \frac{20}{2} = 10\text{ m}.$



The area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10\times(10-8)\times(10-5)\times(10-7)}\ \text{m}^2$

$= \sqrt{10\times2\times5\times3}\ \text{m}^2 = 10\sqrt{3}\ \text{m}^2$

Now, AD is perpendicular to BC.

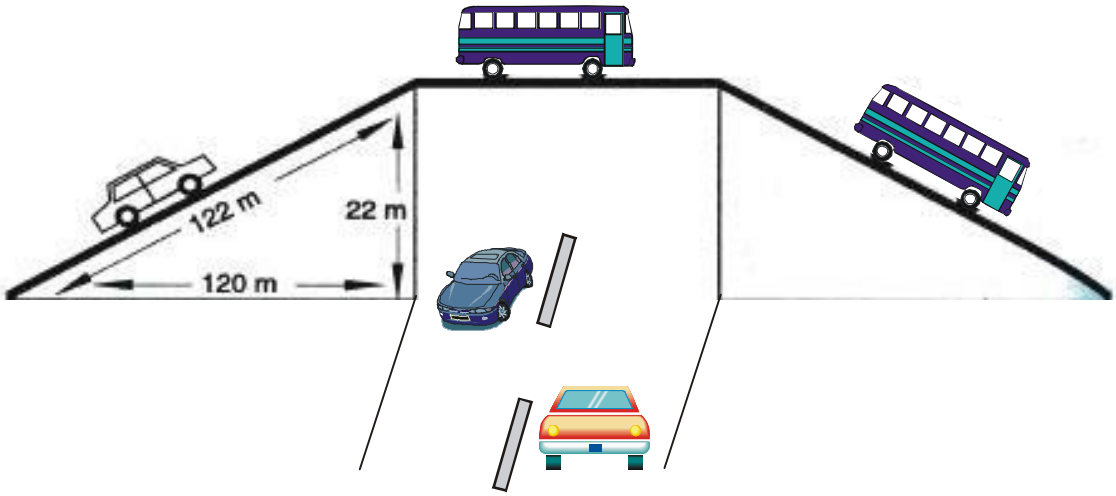
$\Rightarrow \frac{1}{2}\times BC\times AD = 10\sqrt{3}$

$\Rightarrow \frac{1}{2}\times 8\times AD = 10\sqrt{3}$

$\Rightarrow AD = \frac{10\sqrt{3}}{4}\text{m} = \frac{5\sqrt{3}}{2}\text{m}$

Hence, the distance between the trees at A and D is $\frac{5\sqrt{3}}{2}$ m.

Ex.9 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22m and 120 m. The advertisements yield an earning of Rs. 5000 per m^2 per year. A company hired both walls for 3 months. How much rent did it pay?



Sol. The lengths of the sides of the walls are 122 m, 22 m and 120 m.

We have, $122^2 = 120^2 + 20^2 = 14884$

So, walls are in the form of a right triangle.

$\therefore$ Area of two walls = 2 x $\left(\frac{1}{2}\times\text{Base}\times\text{Height}\right)$

$\Rightarrow$ Area of two walls = 2 x $\left(\frac{1}{2}\times120\times22\right)$ = 2640 m^2

We have,

Yearly rent = Rs 5000 per m^2

$\therefore$ Monthly rent = Rs $\left(\frac{5000}{12}\right)$ per m^2

Hence, rent paid by the company for 3 months = Rs $\left(\frac{5000}{12}\times 3\times 2640\right)$ = Rs 3300000.

Ex.10 A traffic signal, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board ?

Sol. (i) Let 2s be the perimeter of the signal board. Then, we have

$2s = a + a + a = 3a \Rightarrow s = \frac{3}{2}a$ [where a is the side of an equilateral triangle]

Let Δ be the area of an equilateral triangle

$\therefore$A of $\Delta = \sqrt{s(s-a)(s-a)(s-a)} = \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)} = \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4}.a^2$.

(ii) If perimeter $\Rightarrow$ 180 = a + a + a

$= 180 = 3a \Rightarrow a = \frac{180}{3} = 60$ cm

and each side of an equilateral Δ(a) = 60 cm

$\therefore$ Area of an equilateral $\Delta = \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3}$ cm^2.

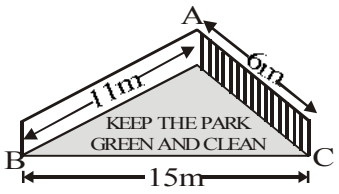
Ex.11 There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" as shown in the figure. If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

Sol. Since the side wall is in the trianglular form with sides a = 15 m, b = 6 m and c = 11 m.

$\therefore$ $2s = a + b + c = (15 + 6 + 11) = 32$

$\Rightarrow$ $s = \frac{32}{2} = 16$ m

$\therefore$ $s - a = 16 - 15 = 1$ m, $s - b = 16 - 6 = 10$ m, $s - c = 16 - 11 = 5$ m



Hence, area to be painted in colour = Area of the side wall

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16\times 1\times 10\times 5} = \sqrt{4\times 4\times 2\times 5\times 5}$

$= 4\times 5\sqrt{2}$ $m^2 = 20\sqrt{2}$ m^2.

Ex.12 An isoscles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol. Area of an isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$ with equal side 'a' and base b.

$\therefore$ a = 12 cm $\Rightarrow$ 2a + b = 30 $\Rightarrow$ 2 × 12 + b = 30, $\Rightarrow$ 24 + b = 30 $\therefore$ b = 30 − 24 = 6 cm

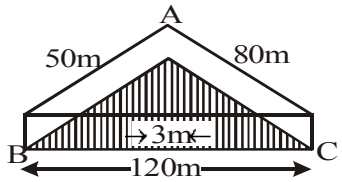
$\therefore$ Area of an isosceles triangle

$= \frac{6}{4}\sqrt{4(12)^2 - (6)^2} = \frac{6}{4}\sqrt{4\times 144 - 36} = \frac{6}{4}\sqrt{576 - 36} = \frac{6}{4}\sqrt{540} = \frac{6}{4}\sqrt{2^2\times 3^2\times 3\times 5} = 6\sqrt{15}$ cm^2

Ex.13 A triangular park ABC has sides 120 m, 80 m and 50 m shown in the figure. A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant ? Find the cost of fencing it with barbed wire at the rate of Rs. 20 per metre leaving a space 3m wide for a gate on one side.

Sol. For area of the park, we have

$s = \frac{120 + 80 + 50}{2} - \frac{250}{2} = 125$ m



Now, $s - a = (125 - 120)$ m $= 5$m

$s - b = (125 - 80)$ m $= 45$ m

$s - c = (125 - 50)$ m $= 75$ m

Therefore, area of the park = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}$ m^2 $= \sqrt{25 \times 25 \times 3 \times 15 \times 15 \times 5}$ m^2 $= 375\sqrt{15}$ m^2.

Also, perimeter of the park = AB + BC + CA = 250 m

Therefore, length of the wire needed for fencing = 250 m – 3 m (to be left for gate) = 247 m.

And the cost of fencing = Rs. 20 × 247 = Rs. 4940.

Ex.14 Find the base of an isosceles triangle whose area is 12 cm^2 and one equal sides is 5 cm.

Sol. Here equal sides : a = 5 cm, b = ?, Area = 12 cm^2

Area of an isosceles triangle = 12 cm^2

$\Rightarrow \frac{b}{4}\sqrt{4a^2 - b^2} = 12 \Rightarrow \frac{b}{4}\sqrt{4 \times 5^2 - b^2} = 12 \Rightarrow b\sqrt{100 - b^2} = 12 \times 4$

On squaring both sides, we get

$b^2 (100 - b^2) = 2304$ or $b^4 - 100b^2 + 2304 = 0$

or $b^4 - 64b^2 - 36b^2 + 2304 = 0$ or $b^2(b^2 - 64) - 36(b^2 - 64) = 0$

or $(b^2 - 36)(b^2 - 64) = 0$

$\Rightarrow$ either $b^2 = 36$ or $b^2 = 64$

$\Rightarrow$ $b = \pm 6$ $\Rightarrow$ $b = \pm 8$

Neglecting the negative sign as length cannot be –ve

$\therefore$ Base(b) = 8 cm or 6 cm

Ex.15 The perimeter of a right triangle is 144 cm and its hypotenuse measures 65 cm. Find the lengths of other sides and calculate its area. Verify the result using Heron's formula.

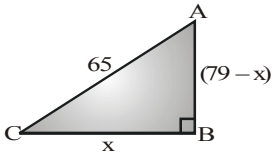
Sol. Perimeter of a right triangle = 144 cm ; Hypotenuse = 65 cm

Sum of the other two sides = 144 – 65 = 79 cm

Let one side be x, then the other side is (79 – x) cm.

In a right angle $\triangle$ABC, by pythagorus we have $AC^2 = AB^2 + BC^2$

$(65)^2 = (79 - x)^2 + x^2$



$4225 = 6241 + x^2 - 158x + x^2 \Rightarrow 4225 = 6241 + 2x^2 - 158x$

$2x^2 - 158x + 6241 - 4225 = 0 \Rightarrow 2x^2 - 158x + 2016 = 0$

$x^2 - 79x + 1008 = 0 \Rightarrow x^2 - 16x - 63x + 1008 = 0$

$x(x - 16) - 63(x - 16) = 0 \Rightarrow (x - 16)(x - 63) = 0$

$\Rightarrow$ either $x = 16$ cm or $x = 63$ cm

(i) When $x = 16$ cm $\Rightarrow$ BC = 16 cm and AB = 79 – 16 = 63 cm

(ii) When $x = 63$ cm $\Rightarrow$ BC = 63 cm and AB = 79 – 63 = 16 cm

Hence, the three sides of the triangle are 65 cm, 63 cm and 16 cm

$\therefore$ Area of right triangle $= \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 16 \times 63 =$ **504 cm²**

Verification by Heron's formula, we have

$a = 63$ cm, $b = 65$ cm, $c = 16$ cm

$\Rightarrow s = \frac{63 + 15 + 16}{2} = 72$ cm

$\therefore s - a = 72 - 63 = 9$ cm, $s - b = 72 - 65 = 7$ cm, $s - c = 72 - 16 = 56$ cm

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{72 \times 9 \times 7 \times 56} = \sqrt{9 \times 8 \times 9 \times 7 \times 7 \times 8} = 9 \times 7 \times 8 =$ **504 cm²**

Ex.16 One side of an equilateral triangle measures 8 cm. Find the area using Heron's formula. What is its altitude ?

Sol. Each side of an equilateral triangle = 8 cm

Here, $a = 8$ cm, $b = 8$ cm, $c = 8$ cm

$\therefore s = \frac{a+b+c}{2} = \frac{8+8+8}{2} = \frac{24}{2} = 12$ cm, $s - a = s - b = s - c = 12 - 8 = 4$ cm

$\therefore$ **(i) Area of the triangle is**

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{12(12-8)(12-8)(12-8)}$ cm² $= \sqrt{12 \times 4 \times 4 \times 4}$ cm² $= \sqrt{3 \times 4 \times 4 \times 4 \times 4}$ cm²

$= \sqrt{3} \times 4 \times 4$ cm² $= 16\sqrt{3}$ cm².

and **(ii) Altitude of an equilateral triangle (h)**

$h = \frac{\sqrt{3}}{2} \times$ (side) $= \frac{\sqrt{3}}{2} \times 8$ cm $= 4\sqrt{3}$ cm.

Ex.17 Find the area of an equilateral triangle whose one side x cm.

Sol. $2s = x + x + x, \Rightarrow 2s = 3x \Rightarrow s = \frac{3x}{2}$ $\therefore$ $s - a = s - b = s - c = \frac{3}{2}x - x = \frac{1}{2}x$

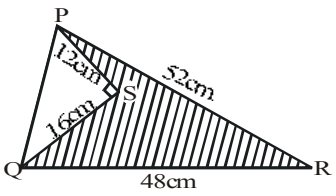
$\therefore$ Area of an equilateral $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{3}{2}x \times \frac{1}{2}x \times \frac{1}{2}x \times \frac{1}{2}x} = \frac{\sqrt{3}}{4} \times x^2 = \frac{\sqrt{3}}{4}.(\text{side})^2$

Ex.18 Calculate the area of the shaded portion of the Δ as shown in figure.

Sol. In right ΔPSQ, by Pythagoras

Theorem $PQ^2 = PS^2 + SQ^2 = (12)^2 + (16)^2 = 144 + 256 = 400$ cm

$\Rightarrow$ $PQ = \sqrt{400}$ cm $= 20$ cm



Now, a = 20 cm, b = 48 cm and c = 52 cm

$\therefore \quad s = \frac{a+b+c}{2} = \frac{20\text{cm} + 48\text{cm} + 52\text{cm}}{2} = \frac{120\text{cm}}{2} = 60\text{cm}$

Area of $\Delta PQR = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-48)(60-52)}$

$= \sqrt{60 \times 40 \times 12 \times 8} = \sqrt{6 \times 10 \times 10 \times 4 \times 6 \times 2 \times 8}$

$= 6 \times 10 \times 8 = 480 \text{ cm}^2$.

and area of $\Delta PQS = \frac{1}{2} \times PS \times QS = \frac{1}{2} \times 12 \times 16 = 96 \text{ cm}^2$

$\Rightarrow$ area of the shaded portion of the triangle = 480 – 96 = 384 cm^2.

Ex.19 The sides of triangular plate are 8 cm, 15 cm and 17 cm. If its weight is 96 gram, find the weight of the plate per square cm.

Sol. Here a = 15 cm, b = 17 cm, c = 8 cm.

Since $(17)^2 = 289$ and $(15)^2 + (8)^2 = 225 + 64 = 289$

$\therefore$ This is a right triangle with sides 8 cm and 15 cm

$\therefore$ Area of right triangle $= \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$

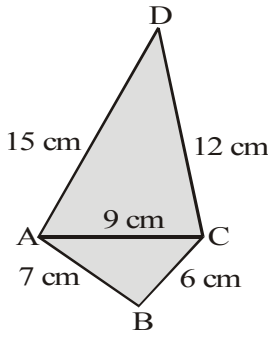
Weight of triangle plate = 96 gram

$\therefore$ Weight per square cm $= \frac{96}{60} = 1.6$ gm.

Ex.20 Find the area of the quadrilateral ABCD, in which AB = 7 cm, BC = 6 cm, CD= 12 cm, DA = 15 cm and AC = 9 cm.

Sol. The diagonal AC divides the quadrilateral ABCD into two triangles ABC and ACD.

$\therefore$ Area of quad. ABCD = Area of $\triangle ABC$ + Area of $\triangle ACD$,



For ΔABC, we have

Semiperimeter $s = \frac{6+7+9}{2} = 11$ cm

$\therefore$ Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ A_1 = Area of $\Delta ABC = \sqrt{11(11-6)(11-7)(11-9)}$

$\Rightarrow$ A_1 = Area of $\Delta ABC = \sqrt{11 \times 5 \times 4 \times 2} = \sqrt{440}$ sq. cm

$\Rightarrow$ A_1 = Area of $\Delta ABC = 20.98 \text{ cm}^2$

For ΔACD, we have $s = \frac{9+12+15}{2} = 18$ cm

Area of $\triangle ACD = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ A_2 = Area of $\Delta ACD = \sqrt{18(18-9)(18-12)(18-15)}$

$\Rightarrow$ A_2 = Area of $\Delta ACD = \sqrt{18 \times 9 \times 6 \times 3}$

$\Rightarrow$ A_2 = Area of ΔACD

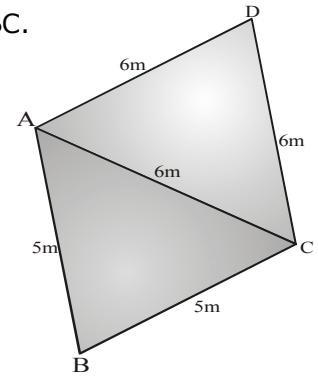
$= \sqrt{2 \times 9 \times 9 \times 2 \times 3 \times 3} = \sqrt{9^2 \times 2^2 \times 3^2}$

$= 9 \times 2 \times 3 = 54 \text{ cm}^2$

Hence, Area of quad. ABCD = $A_1 + A_2 = (20.98 + 54) \text{ cm}^2 = 74.98 \text{ cm}^2$

Ex.21 Prove that the area of the quadrilateral ABCD is $3(4+3\sqrt{3})\,m^2$, if AB = 5 m, BC = 5 m, CD = 6 m, AD = 6 m, and diagonal AC = 6 m.

Sol. Diagonal AC divides the quadrilateral ABCD into two triangles ΔACD and ΔABC.



For ΔACD, side are 6m, 6 m and 6 m.

Semiperimeter, s = $\frac{6m+6m+6m}{2}$ = 9 m

$\therefore$ Area of ΔACD = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{9\times(9-6)(9-6)\times(9-6)}$ m^2

= $\sqrt{9\times3\times3\times3}$ = $9\sqrt{3}$ m^2

For ΔABC, side are 5 m, 5 m and 6 m.

Semiperimeter, s = $\frac{5m+5m+6m}{2}$ = 8 m

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{8(8-5)(8-5)(8-6)}$ = $\sqrt{8\times3\times3\times2}\,m^2$ = $\sqrt{16\times9}$ m^2 = 12 m^2

Thus, the area of the quadrilateral ABCD = (12 + 9$\sqrt{3}$) m^2 = 3(4 + 3$\sqrt{3}$) m^2.

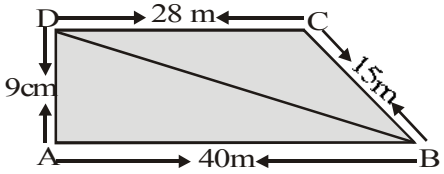
Ex.22 In fig. ABCD is a field in the form of a quadrilateral whose sides are indicated in the figure. If $\angle$DAB = 90°, find the area of the field.

Sol. Clearly, ΔDAB is a right-angled triangle. Therefore,

$DB^2 = DA^2 + AB^2$ [Using Pythagoras Theorem]

$\Rightarrow$ $DB^2 = 9^2 + 40^2$

$\Rightarrow$ DB = $\sqrt{81+1600}$ m = $\sqrt{1681}$ = 41m



For ΔDAB, we have s = $\frac{9+40+41}{2}-\frac{90}{2}$ = 45 m

Therefore, Area of ΔDAB = $\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ A_1 = Area of ΔDAB = $\sqrt{45\times(45-9)\times(45-40)\times(45-41)}$ m^2

$\Rightarrow$ A_1 = Area of ΔDAB = $\sqrt{45\times36\times5\times4}$ m^2

$\Rightarrow$ A_1 = Area of ΔDAB = $\sqrt{5\times9\times36\times5\times4}\,m^2$ = $\sqrt{5^2\times3^2\times6^2\times2^2}\,m^2$

$\Rightarrow$ A_1 = Area of ΔDAB = (5 × 3 × 6 × 2) m^2 = 180 m^2

For ΔDCB, we have s = $\frac{28+15+41}{2}=\frac{84}{2}$ = 42 m.

$\Rightarrow$ A_2 = Area of ΔDCB = $\sqrt{42\times(42-28)\times(42-15)\times(42-41)}$ m^2

$\Rightarrow$ A_2 = Area of ΔDCB = $\sqrt{42\times14\times27\times1}$ m^2

$\Rightarrow$ A_2 = Area of ΔDCB = $\sqrt{7\times2\times3\times7\times2\times3\times3\times3}$ m^2

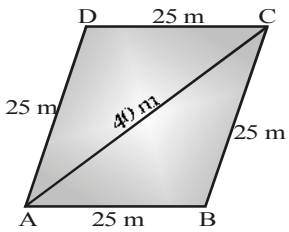
$\Rightarrow$ A_2 = Area of ΔDCB = $\sqrt{7^2\times2^2\times3^4}$ m^2

$\Rightarrow$ A_2 = Area of ΔDCB = (7 × 2 × 3^2) m^2 = 126 m^2

Hence, Area of the field = A_1 + A_2 = (180 + 126) m^2 = 306 m^2

Ex.23 A rhombus has perimeter 100 m and one of its diagonal is 40 m. Find the area of the rhombus.

Sol. ABCD is the rhombus having perimeter = 100 m and AC = 40 m.



Now, we have AB = BC = CD = AD = $\frac{100}{4}$ = 25 m

We know that, ar (ΔABC) = ar (ΔADC)

Sides of ΔABC are 25 m, 25 m and 40 m.

Semi perimeter of ΔABC (s) = $\frac{25\text{m} + 25\text{m} + 40\text{m}}{2}$ = 45 m.

The area of ΔABC = $\sqrt{45 \times (45-25) \times (45-25) \times (45-40)}$ m^2 = $\sqrt{45 \times 20 \times 20 \times 5}$ m^2

= $\sqrt{9 \times 25 \times 20 \times 20}$ = 3 × 5 × 20 m^2 = 300 m^2

Also, we have area of ΔADC = 300 m^2.

Hence, the area of the rhombus ABCD = ar (ΔABC) + ar (ΔADC) = 300 m^2 + 300 m^2 = 600 m^2.

Ex.24 Find the area of a trapezium whose parallel sides 25 cm, 13 cm and other sides are 15 cm and 15 cm.

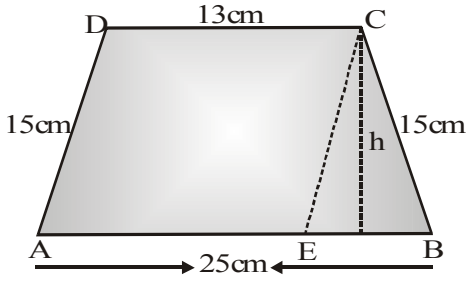
Sol. Let ABCD be the given trapezium in which AB = 25 cm, CD = 13 cm, BC = 15 cm and AD = 15 cm.

Draw CE||AD.

Now, ADCE is a parallelogram in which AD||CE and AE||CD.

AE = DC = 13 cm and BE = AB - AE = 25 - 13 = 12 cm

In ΔBCE, we have



s = $\frac{15+15+12}{2}$ = 21 cm

$\therefore$ Area of ΔBCE = $\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{21(21-15)(21-15)(21-12)}$

$\Rightarrow$ Area of ΔBCE = $\sqrt{21 \times 6 \times 6 \times 9}$ = $18\sqrt{21}$ sq. cm ...(i)

Let h be the height of ΔBCE, then

Area of ΔBCE = $\frac{1}{2}$ (Base × Height) = $\frac{1}{2}$ × 12 × h = 6h ...(ii)

From (i) and (ii), we have,

6h = $18\sqrt{21}$ $\Rightarrow$ h = $3\sqrt{21}$ cm

Clearly, the height of trapezium ABCD is same as that of ΔBCE.

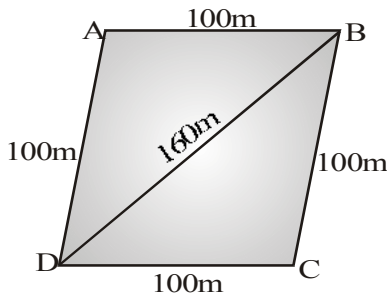
$\therefore$ Area of trapezium = $\frac{1}{2}$ (AB + CD) × h

$\Rightarrow$ Area of trapezium = $\frac{1}{2}$ (25 + 13) × $3\sqrt{21}$ cm^2 = $57\sqrt{21}$ cm^2

Ex.25 Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops to suffice the needs of their family. She divided the land in two equals parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get?

Sol. Let ABCD be the field which is divided by the diagonal BD = 160 m into two equal parts.

Since ABCD is a rhombus of perimeter 400 m. Therefore,



$$AB = BC = CD = DA = \frac{400}{4} \text{ m} = 100 \text{ m}$$

Let s be the semi-perimeter of $\triangle ABCD$.

Then, $$s = \frac{BC + CD + BD}{2} = \frac{100 + 100 + 160}{2} \text{ m} = 180 \text{ m}$$

$\therefore$ Area of $\triangle BCD$

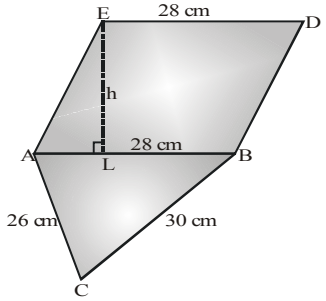
$$= \sqrt{180 \times (180 - 100) \times (180 - 100) \times (180 - 160)} \text{ m}^2$$

$$= \sqrt{180 \times 80 \times 80 \times 20} \text{ m}^2 = 4800 \text{ m}^2$$

Hence, each of the two children will get an area of 4800 m^2.

Ex.26 A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Sol. Semiperimeter of $\triangle ABC$



$$s = \frac{26 + 28 + 30}{2} = 42 \text{ cm}$$

$$s - a = 42 - 26 = 16 \text{ cm}$$

$$s - b = 42 - 28 = 14 \text{ cm}$$

$$s - c = 42 - 30 = 12 \text{ cm}$$

Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42 \times 16 \times 14 \times 12} \text{ cm}^2 = \sqrt{14 \times 3 \times 4 \times 4 \times 14 \times 4 \times 3} \text{ cm}^2$$

$$= 14 \times 4 \times 3 \times 2 \text{ cm}^2 = 336 \text{ cm}^2$$

$\therefore$ Area of parallelogram = Area of triangle [Given]

$$h \times AB = 336$$

$$h \times 28 = 336 \text{ cm}^2$$

$$h = \frac{336}{28} = 12 \text{ cm}$$

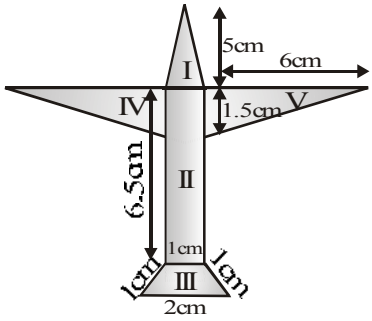
Ex.27 Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.

Sol. **Area of I in figure**

It is triangle part and its sides are 5 cm, 5 cm, 1 cm. Here, semiperimeter of the triangle

$= \frac{5\,cm + 5\,cm + 1\,cm}{2} = \frac{11}{2}\,cm$

Area of part I $= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}\,cm^2 = \frac{3}{4}\sqrt{11}\,cm^2$



$= \frac{3}{4} \times 3.31\ cm^2 = 2.482$ (approx)

Area of II in figure = area of rectangle = L × B = 6.5 × 1 = 65 cm^2

Area of III in figure

Area of $\Delta BEC = \frac{\sqrt{3}}{4}(1)^2\,cm^2 = \frac{\sqrt{3}}{4}cm^2$

Let h be the height of the ΔBEC

$\frac{1}{2} \times BE \times h = \frac{\sqrt{3}}{4} \Rightarrow \frac{1}{2} \times 1 \times h = \frac{\sqrt{3}}{4} \Rightarrow h = \frac{\sqrt{3}}{2}cm$

Area of Region III $= \frac{1}{2}(1 + 2) \times \frac{\sqrt{3}}{2}cm^2 = \frac{3}{4}\sqrt{3}\ cm^2$

$= \frac{3}{4} \times 1.732\ cm^2 = 1.3\ cm^2$ (approx)

Area of IV in figure $= \frac{1}{2} \times 6 \times \frac{3}{2}cm^2 = \frac{9}{2}cm^2$

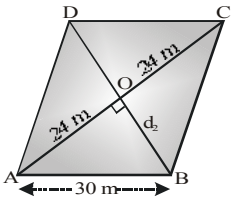
Area of V in figure $= \frac{1}{2} \times 6 \times \frac{3}{2}cm^2 = \frac{9}{2}cm^2$

Total area of the paper used = 2.482 cm^2 + 6.5 cm^2 + 1.3 cm^2 + $\frac{9}{2}cm^2$ + $\frac{9}{2}cm^2$

= (10.282 + 9) cm^2 (approx.) = 19.282 cm^2 (approx.) = 19.3 cm^2 (approx.)

Ex.28 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 cm, how much area of grass field will each cow be getting ?

Sol. Since the diagonals of a Rhombus bisect each other at right angles



$\therefore$ OB $= \sqrt{(30)^2 - (24)^2} = \sqrt{324} = 18$ cm

$\Rightarrow$ diagonal of d_2 = 2 × OB = 2 × 18 = 36 cm

$\therefore$ Area of a Rhombus $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 48 \times 36 = 864\ m^2$

$[d_1 = 48$ cm (given)]

Total area of grass field for 18 cows = 864 m^2

Area of grass grazed by each cow $= \frac{864}{18} = 48\ m^2$.

Ex.29 An umbrealla is made by stitching 10 triangular pieces of cloth of two different colour (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella ?

Sol. Sides of triangular piece of coloured cloth are 20 cm, 50 cm, 50 cm

Semiperimeter, $s = \frac{20+50+50}{2} = \frac{120}{2} = 60$ cm

Then, area of one triangular piece

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)} = \sqrt{60\times40\times10\times10} = \sqrt{10\times6\times10\times4\times10\times10}$

$= 10 \times 10 \times 2\sqrt{6}$ cm^2 $= 200\sqrt{6}$ cm^2

There are 5 triangular pieces of one colour and 5 of the other colours

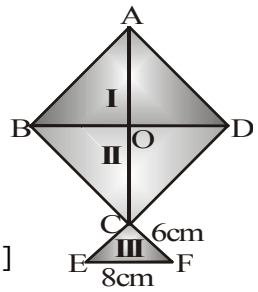
Then, total area of cloth of each colour = $5 \times 200\sqrt{6}$ cm^2 $= 1000\sqrt{6}$ cm^2

Ex.30 A kite in the shape of a square is made of three different shades marked as I, II and III as shown in figure. How much paper of each shade has been used in it, if each diagonal of this kite is 32 cm and lower potion has the sides 6 cm, 6 cm and 8 cm ?

Sol. Each diagonal of the square = 32 cm

Area of I = $\frac{1}{2} \times 32 \times 16 = 256$ cm^2

Area of II = $\frac{1}{2} \times 32 \times 16 = 256$ cm^2

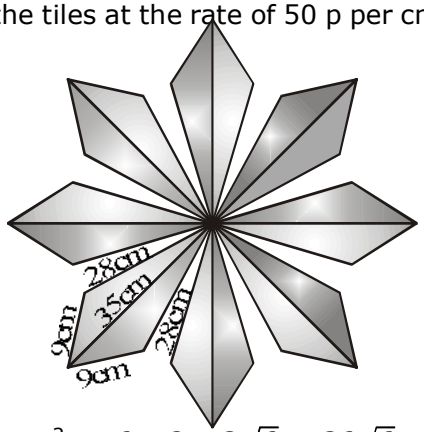


Area of III = $\sqrt{10(10-6)(10-6)(10-8)}$ [$\because s = \frac{6+6+8}{2} = \frac{20}{2} = 10$ cm]

$= \sqrt{10\times4\times4\times2} = 8\sqrt{5}$ cm^2 = 17.92 cm^2

Ex.31 A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm shown in the figure. Find the cost of polishing the tiles at the rate of 50 p per cm^2.

Sol. Sides of triangular tiles are 9 cm, 28 cm and 35 cm



Its semiperimeter, $s = \frac{35+28+9}{2} = \frac{72}{2} = 36$ cm

s – a = 36 – 35 = 1 cm

s – b = 36 – 28 = 8 cm

s – c = 36 – 9 = 27 cm

Area of one triangular tile

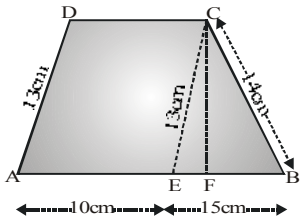
$= \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{36\times1\times8\times27}$ cm^2 $= \sqrt{36\times4\times2\times3\times9}$ cm^2 $= 6 \times 2 \times 3\sqrt{6} = 36\sqrt{6}$ cm^2

Total area of 16 tiles $= 16 \times 36\sqrt{6}$ cm^2

$= 576\sqrt{6}$ cm^2 $= 576 \times 2.45$ cm^2 (approx.) $= 1411.20$ cm^2 (approx.)

Total cost of polishing at the rate of 50p per cm^2 = Rs. $1411.20 \times \frac{50}{100}$ = Rs. 705.60

Ex.32 A field is in the shape of a tapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.



Sol. Through C draw CE || DA

Draw CF ⊥ AB

In ΔBCE, we have

$s=\frac{15+14+13}{2}=21$ m

s – a = 21 – 15 = 6 cm, s – b = 21 – 14 = 7 cm, s – c = 21 – 13 = 8 cm

Area of ΔBCE = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{21\times6\times7\times8}$ cm^2 = 7 × 4 × 3 = 84 cm^2

Now, area of ΔBCE = 84 m^2

$\Rightarrow$ $\frac{1}{2}$ × Base × Altitude = 84 m^2 $\Rightarrow$ $\frac{1}{2}$ × 15 × h = 84 $\Rightarrow$ h = $\frac{84\times2}{15}$m

$\Rightarrow$ Distance between parallel sides of trapezium = $\frac{168}{15}$m

Area of parallelogram, AECD = base × height = 10 × $\frac{168}{15}$ = 56 × 2 = 112 m^2

$\therefore$ Area of trapezium ABCD = Area of parallelogram

AECD + Area of ΔBCE = 112 + 84 = 196 m^2.

EXERCISE - I UNSOLVED PROBLEM

Q.1 Find the area of a triangle, two of whose sides are 18 cm and 10 cm and perimeter is 42 cm.

Q.2 Find the area of a triangle two of whose sides are 8 cm and 11 cm and perimeter is 32 cm. Also find the altitude corresponding to the base 11 cm.

Q.3 Sides of triangle ABC are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.

Q.4 An isosceles triangle has perimeter 30 cm and each of the equal is 12 cm. Find the area of the triangle

Q.5 A triangular field has dimensions 24 m, 7 m and 25 m. It has a road running around it so that the triangular region including road has dimensions 26 m, 9 m and 27 m. Find the cost of fitting tiles on the road at the rate of Rs. 80 per m^2. (use $\sqrt{34100} = 58.40$)

Q.6 The perimeter of a right angled triangle is 60 cm. Its hypotenuse is 26 cm, find the other two sides and the area of triangle. Verify the result using Heron's formula.

Q.7 Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Q.8 A park, in the shape of a quadrilateral ABCD has $\angle C = 90^\circ$, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Q.9 If the perimeter of a triangle is 300 cm and its sides are in the ratio 5 : 12 : 13, find area of the triangle.

Q.10 Find the area of a ABCD whose sides in metres are 9, 40, 28 and 15 respectively and the angle between first two sides is a right angle.

Q.11 Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 6 cm, DA = 5 cm and diagonal AC = 5 cm.

Q.12 Determine the diagonal of a quadrilateral whose area is 495 cm^2, where perpendiculars from the opposite vertices on the diagonal are 11 cm and 19 cm respectively.

Q.13 Find the ratio of the area of a square to that of the square drawn on its diagonal.

Q.14 The adjacent sides of a parallelogram are 24 cm and 32 cm. If the distance between the longer sides is 17.4 cm, determine the distance between the shorter sides.

Q.15 Find percentage increase in the area of triangle if its sides are doubled.

Q.16 The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Q.17 If the area of an equilateral triangle is $81\sqrt{3}$ cm^2, find its height.

Q.18 If the area of an equilateral triangle $36\sqrt{3}$ cm^2, find its perimeter.

Q.19 Find the area of rhombus, if perimeter is 120 m and longer diagonal is 48 m.

Q.20 Using Heron's formula find the area of an isosceles triangle whose one of the equal sides is 16 cm and third side is 10 cm.

Q.21 The perimeter of a right triangle is 144 cm and its hypotenuse measures 65 cm. Find the lengths of other sides and calculate its area. Verify the result using Hero's formula.

Q.22 The base of an isosceles triangle is 14 cm and one of its equal sides is 12 cm. Find its area using Hero's formula.

Q.23 The sides of a triangle are of lengths 10 cm, 15 cm and 15 cm. Find the length of the altitude drawn on the side with length 15 cm.

Q.24 An isosceles right triangle has an area 200 cm^2. What is the length of its hypotenuse?

Q.25 The perimeter of a right triangles is 12 cm and its hypotenuse is of length 5 cm. Find the other two sides and calculate its area.

Q.26 The sides of a triangle are of lengths 8 cm, 15 cm and 17 cm. Find the length of the altitude drawn on the side with length 17 cm.

Q.27 The base of an isosceles triangle measures 24 cm and its area is 192 cm^2. Find its perimeter.

Q.28 Find the area of an isosceles right-angled triangle, each of whose equal sides measures 10 cm. (Take $\sqrt{2}$ = 1.414)

Q.29 Find the base of an isosceles triangle whose area is 12 cm^2 and one of the equal sides is 5 cm.

Q.30 The lengths of the sides of triangle ABC are in the ratio 4 : 3 : 5, and its perimeter is 144 cm. Find the height corresponding to the longest side.

ANSWER KEY

1. $21\sqrt{11}$ cm^2 **2.** $8\sqrt{30}$ cm^2 ; $\frac{16\sqrt{30}}{11}$ cm

3. 9000 cm^2 **4.** $9\sqrt{15}$ cm^2

5. Rs. 2624

6. Sides = 24cm, 10 cm Area = 120 cm^2

7. 15.16 cm^2 . **8.** 65.496 m^2

9. 3000 cm^2 **10.** 306 m^2

11. 18 cm^2 **12.** 33 cm

13. ratio is 1 : 2 **14.** 23.2 cm

15. 300 % **16.** $20\sqrt{30}$ cm^2

17. $9\sqrt{3}$ cm **18.** 36 cm

19. Area = 864 m^2 **20.** $5\sqrt{231}$ cm^2

21. 504 cm^2 **22.** $7\sqrt{95}$ cm^2

23. 9.42 cm **24.** $20\sqrt{2}$ cm

25. 3 cm, 4 cm; 6 cm^2

26. $7\frac{1}{17}$ cm **27.** 64 cm

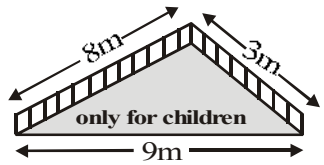
28. 50 cm^2 **29.** 8 cm or 6 cm

30. 28.8 cm

EXERCISE – II SCHOOL EXAM/BOARD

Q.1 A triangular park in a city has dimensions 100 m × 90 m × 110 m. A contract is given to a company for planting grass in the park at the rate of Rs. 4000 per hectare. Find the amount to be paid to the company. (Take $\sqrt{2}$ = 1.414) (one hectare = 10,000 m^2)

Q.2 There is a slide in a children park. The front side of the slide has ben painted and a message "ONLY FOR CHILDREN" is written on it as shown in figure. If the sides of the triangular front wall of the slide are 9 m, 8 m and 3 m, then find the area which is painted in colour.



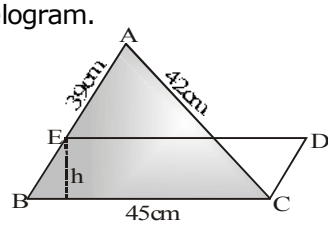
Q.3 The perimeter of a triangular park is 180 m and its sides are in the ratio 5 : 6 : 7. Find the area of the park.

Q.4 A triangle has sides 35 mm, 54 mm and 61 mm long. What is its area. Find also the smallest altitude of the triangle.

Q.5 The perimeter of a right triangle is 12 cm and its hypotenuse is of length 5 cm. Find the other two sides and calculate its area. Verify the result using Heron's Formula.

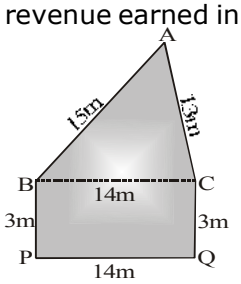
Q.6 Using Heron's Formula, find the area of an isosceles triangle, the measure of one of its equal sides being a units and the third side 2b units.

Q.7 The sides of a triangle are 39 cm, 42 cm and 45 cm. A parallelogram stands on the greatest side of the triangle and has the same area as that of the triangle. Find the height of the parallelogram.



Q.8 From a point in the interior of an equilateral triangle perpendiculars drawn to the three sides are 8 cm, 10 cm and 11 cm respectively. Find the area of the triangle to the nearest cm. (use $\sqrt{3}$ = 1.73)

Q.9 A municipal corporation wall on road side has dimensions as shown in fig. The wall is to be used for advertisements and it yields an earning of Rs. 400 per m^2 in a year. Find the total amount of revenue earned in a year.



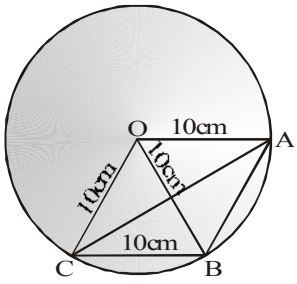
Q.10 ABCD is a quadrilateral such that AB = 5 cm, BC = 4 cm, CD = 7cm, AD = 6 cm and diagonal BD = 5 cm. Prove that the area of the quadrilateral ABCD is $4\left(3+\sqrt{6}\right)cm^2$.

Q.11 Find the area of the quadrilateral ABCD in which AB = 7 cm, BC = 6 cm, CD = 12 cm, DA = 15 cm and AC = 9 cm. $\left(\text{Take } \sqrt{110} = 10.5 \text{ approx.}\right)$

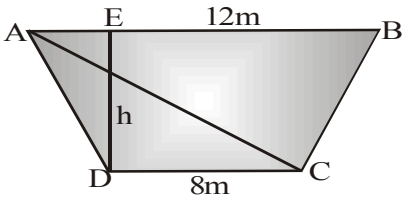
Q.12 A rhombus has perimeter 64 m and one of the diagonals is 22 m. Prove that the area of the rhombus is $66\sqrt{15}\ m^2$

Q.13 ABCD is a trapezium in which AB $\bar{}$ CD ; BC and AD are non-parallel sides. It is given that AB = 75 cm, BC = 42 cm, CD = 30 cm and AD = 39 cm. Find the area of the trapezium.

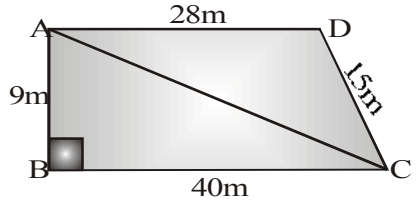
Q.14 OABC is a rhombus whose three vertices A, B and C lie on a circle with centre O. If the radius of the circle is 10 cm, find the area of the rhombus.



Q.15 The cross-section of a canal is in the shape of a trapezium. If the canal is 12 m wide at the top and 8 m wide at the bottom and the area of its cross-section is 84 m^2, determine its depth.



Q.16 Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups. One group walked through the lanes AB, BC and CA ; while the other through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If AB = 9 m, BC = 40 m. CD = 15 m, DA = 28 m and $\angle B = 90°$, which group cleaned more area and by how much? Find the total area cleaned by the students.



Q.17 Find the perimeter of a square, the sum of lengths of whose diagonals is 144 cm.

Q.18 Find the area of a quadrilateral piece of ground one of whose diagonals is 60 metres long and the perpendiculars from the other two vertices are 38 and 22 metres respectively.

Q.19 There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN". If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

Q.20 Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

Q.21 An isosceles triangle has perimeter 30cm and each of the equal sides is 12cm. Find the area of the triangle.

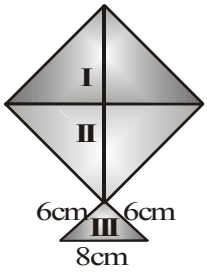
Q.22 A park, in the shape of a quadrilateral ABCD, has $\angle C = 90°$, AB = 9m, BC = 12m, CD = 5m and AD = 8m. How much area does it occupy?

Q.23 Find the area of a quadrilateral ABCD in which AB = 3cm, BC = 4cm, CD = 4cm, DA = 5cm and AC = 5cm

Q.24 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30m and its longer diagonal is 48m, how much area of grass field will each cow be getting?

Q.25 An umbrella is made by stitching 10 triangular pieces of cloth of two different colours, each piece measuring 20cm, 50cm and 50cm. How much cloth of each colour is required for the umbrella?

Q.26 A kite in the shape of a square with a diagonal 32cm and an isosceles triangle of base 8cm and sides 6cm each is to be made of three different shades as shown in fig. How much paper of each shade has been used in it?



ANSWER KEY

1. Rs. 1696.8 **2.** $2\sqrt{35}$ m²

3. $600\sqrt{6}$ m² **4.** $420\sqrt{5}$ mm², $\frac{840\sqrt{5}}{61}$ mm

5. 3 cm. 4 cm; 6 cm² **6.** $b\sqrt{a^2 - b^2}$ sq. units

7. 16.8 cm **8.** 485 cm²

9. Rs. 50400 **11.** 75 cm² (approx)

13. 1764 m² **14.** $50\sqrt{3}$ cm²

15. 8.4 m

16. I group cleaned more area by 54 m² ; 306 m²

17. $144\sqrt{2}$ cm **18.** 1800 m²

19. $20\sqrt{2}$ m² **20.** $21\sqrt{11}$ cm²

21. $9\sqrt{15}$ cm² **22.** 65.5 m² (approx)

23. 15.2 cm² (approx)

24. 48 m²

25. $1000\sqrt{6}$ cm² , $1000\sqrt{6}$ cm²

26. Area of shade I = Area of shade II, = 256 cm² and area of shade III = 17.92 cm²

EXERCISE – III MULTIPLE CHOICE QUESTIONS

Q.1 The height of an equilateral triangle is 6 cm. Its area is

(A) $12\sqrt{3}$ cm^2 (B) $6\sqrt{3}$ cm^2

(C) $12\sqrt{2}$ cm^2 (D) 18 cm^2

Q.2 The lengths of the three sides of a triangular field are 40 m, 24 m and 32 m respectively. The area of the triangle is

(A) 480 m^2 (B) 320 m^2

(C) 384 m^2 (D) 360 m^2

Q.3 The sides of a triangle are in the ratio 5 : 12 : 13 and its perimeter is 150 cm. The area of the triangle is

(A) 375 cm^2 (B) 750 cm^2

(C) 250 cm^2 (D) 500 cm^2

Q.4 The lengths of the three sides of a triangle are 30 cm, 24 cm and 18 cm respectively. The length of the altitude of the triangle corresponding to the smallest side is

(A) 24 cm (B) 18 cm

(C) 30 cm (D) 12 cm

Q.5 The base of an isosceles triangle is 16 cm and its area is 48 cm^2. The perimeter of the triangle is

(A) 41 cm (B) 36 cm

(C) 48 cm (D) 324 cm

Q.6 The area of an equilateral triangle is $36\sqrt{3}$ cm^2. Its perimeter is

(A) 36 cm (B) $12\sqrt{3}$ cm

(C) 24 cm (D) 30 cm

Q.7 Each of the equal sides of an isosceles triangle is 13 cm and its base is 24 cm. The area of the triangle is

(A) 156 cm^2 (B) 78 cm^2

(C) 60 cm^2 (D) 120 cm^2

Q.8 The base of a right triangle is 48 cm and its hpotenuse is 50 cm long. The area of the triangle is

(A) 168 cm^2 (B) 252 cm^2

(C) 336 cm^2 (D) 504 cm^2

Q.9 The area of an equilateral triangle is $81\sqrt{3}$ cm^2. Its height is

(A) $9\sqrt{3}$ cm (B) $6\sqrt{3}$ cm (C) $18\sqrt{3}$ cm (D) 9 cm

Q.10 The difference between the semi-perimeter and the sides of a $\triangle ABC$ are 8 cm, 7 cm and 5 cm respectively. The area of the triangle is

(A) $20\sqrt{7}$ cm^2 (B) $10\sqrt{14}$ cm^2

(C) $20\sqrt{14}$ cm^2 (D) 140 cm^2

Q.11 The perimeter of a triangular field is 144 m and ratio of the sides is 3 : 4 : 5. Then the area of the field is

(A) 864 sq m (B) 764 sq m

(C) 854 sq m (D) 754 sq m

Q.12 One side of an equilateral triangle is 8 cm. Its area is

(A) $16\sqrt{3}$ cm^2 (B) $12\sqrt{3}$ cm^2

(C) $8\sqrt{3}$ cm^2 (D) $4\sqrt{3}$ cm^2

Q.13 The base of an isosceles triangle is 12 cm and its perimeter is 32 cm. Then its area is

(A) 48 sq cm (B) 36 sq cm

(C) 24 sq cm (D) 12 sq cm

Q.14 The area of a triangle whose sides are 13 cm, 14 cm and 15 cm.

(A) 84 sq cm (B) 64 sq cm

(C) 825 sq cm (D) none

Q.15 Two adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm. Then the area of parallelogram is

(A) $5\sqrt{3}$ cm^2 (B) $10\sqrt{3}$ cm^2

(C) $15\sqrt{3}$ cm^2 (D) $20\sqrt{3}$ cm^2

Q.16 Two adjacent sides of a parallelogram are 51 cm and 37 cm. One of its diagonals is 20 cm, then the its area is

(A) 412 cm^2 (B) 512 cm^2

(C) 612 cm^2 (D) 712 cm^2

Q.17 The sides of a triangle are in the ratio of 13 : 14 : 15 and its perimeter is 84 cm. Then the area of the triangle is

(A) 136 cm^2 (B) 236 cm^2

(C) 336 cm^2 (D) 436 cm^2

Q.18 The area of a parallelogram whose diagonals is 6.8 cm and the perpendicular distance of this diagonal from an opposite vertex is 7.5 cm is

(A) 25.5 cm^2 (B) 11.9 cm^2

(C) 12.5 cm^2 (D) 51 cm^2

Q.19 The adjacent sides of a parallelogram are 4 cm and 9 cm. The ratio of its altitudes is

(A) 16 : 81 (B) 9 : 4

(C) 2 : 3 (D) 3 : 2

Q.20 The perimeter of a rhombus is 52 cm and one of its diagonals is 24 cm. The length of the other diagonals is

(A) 24 cm (B) 10 cm

(C) $2\frac{1}{6}$ (D) 12 cm

Q.21 In quadrilateral ABCD given that AB = 7 cm, BC = 12 cm, CD = 12 cm, DA = 9 cm and diagonals AC = 15 cm. It's area is

(A) $(10\sqrt{34}+54)$ sq cm

(B) $(10\sqrt{34}-54)$ sq cm

(C) data insufficient

(D) none of these

Q.22 Adjacent sides of a parallelogram are 5 cm and 3.5 cm. One of its diagonals is 6.5 cm. Then the area of parallelogram is

(A) $8\sqrt{3}$ cm^2 (B) $9\sqrt{3}$ cm^2

(C) $10\sqrt{3}$ cm^2 (D) $12\sqrt{3}$ cm^2

Q.23 The perimeter of a rhombus is 146 cm. One of its diagonals is 55 cm. Then the length of the other diagonal and the area of the rhombus is

(A) 48 cm, 1320 sq cm

(B) 45 cm, 660 sq cm

(C) 27.5 cm, 660 sq cm

(D) none of these

Q.24 In a quadrilateral the sides are 9, 40, 28, 15 units and the angle between first sides is a right angle. The area of quadrilateral is

(A) 106 sq units (B) 206 sq units

(C) 306 sq units (D) 406 sq units

Q.25 In a quarilateral ABCD, AB = 7 cm, BC = 6 cm, CD = 12 cm, DA = 15 cm, AC = 9 cm. Its area is

(A) $(\sqrt{440}+54)$ sq cm

(B) $(\sqrt{440}+44)$ sq cm

(C) $(\sqrt{110}+44)$ sq cm

(D) $(\sqrt{340}+64)$ sq cm

Q.26 The area of a rhombus is 28 cm^2 and one of its diagonals is 4 cm. Its perimeter is

(A) $4\sqrt{53}$ cm (B) 36 cm

(C) $2\sqrt{53}$ cm (D) none

Q.27 The adjacent sides of a parallelogram are 8 cm and 9 cm. The diagonal joining the ends of these sides is 13 cm. Its area is

(A) 72 cm^2 (B) $12\sqrt{35}$ cm^2

(C) $2\sqrt{35}$ cm^2 (D) 150 cm^2

Q.28 The sides of a triangle are 11 cm, 15 cm and 16 cm. The altitude to largest side is

(A) $30\sqrt{7}$ (B) $\frac{15\sqrt{7}}{2}$ cm

(C) $\frac{15\sqrt{7}}{4}$ cm (D) 30 cm

Q.29 The perimeter of a triangular field is 144 m and the ratio of the sides is 3 : 4 : 5. The area of the field is

(A) 864 m^2 (B) 468 m^2

(C) 824 m^2 (D) none

Q.30 If the altitude of an equilateral triangle is $\sqrt{6}$ cm, its area is

(A) $2\sqrt{3}$ cm^2 (B) $2\sqrt{2}$ cm^2

(C) $3\sqrt{3}$ cm^2 (D) $6\sqrt{2}$ cm^2

Q.31 The area of triangle whose sides are 18 cm, 10 cm and 14 cm is

(A) 241 cm^2 (B) $21\sqrt{11}$ cm^2

(C) $21\sqrt{15}$ cm^2 (D) none of these

Q.32 The cost of turfing a triangular field at the rate of Rs. 5 per sq. m is Rs. 1350. If the sides of the field are in the ratio 5 : 12 : 13, then the sides of the field are

(A) 5, 12, 13 (B) 10, 24, 26

(C) 15, 36, 39 (D) none of these

Q.33 From a point in the interior of an equilateral triangle, perpendiculars drawn to the three sides are of length 8 cm, 10 cm and 11 cm respectively. Then the area of triangle is

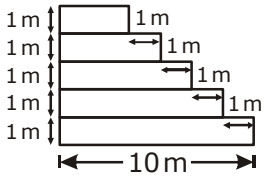
(A) $\frac{841\sqrt{3}}{3}$ cm^2

(B) $841\sqrt{3}$ cm^2

(C) $\frac{841}{3}$ cm^2

(D) cannot be found with given data

Q.34 Area of the given figure is



(A) 50 m^2 (B) 40 m^2

(C) 45 m^2 (D) 48 m^2

Q.35 The area of a rectangular field is 150 sq. units. If its perimeter is 50 units, its dimensions are

(A) 2,75 (B) 3,50

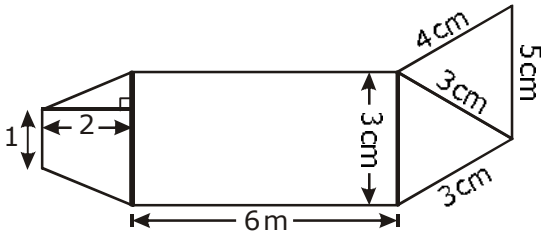
(C) 5,30 (D) 10,15

Q.36 If each side of a rectangle is increased by 50%, its area will increase by

(A) 50% (B) 125%

(C) 100% (D) none of these

Q.37 The area of the given figure is



(A) $24+\frac{9}{4}\sqrt{3}$ (B) $34+\frac{9}{4}\sqrt{3}$

(C) $28+\frac{9}{4}\sqrt{3}$ (D) none of these

Q.38 The perimeter of a right angled triangle is 24 cm and its hypotenuse 10 cm. The area of the triangle is

(A) 240 cm^2 (B) 24 cm^2

(C) 120 cm^2 (D) 48 cm^2

Q.39 The side of a regular hexagon is a. Its area is

(A) $\frac{3\sqrt{3}}{2}a^2$ sq. units (B) $\frac{\sqrt{3}}{2}a^2$ sq. units

(C) $2\sqrt{3}\,a^2$ sq. units (D) $6a^2$ sq. units

Q.40 If every side of a triangle is doubled, the area of the new triangle is k times the area of the old one. k =

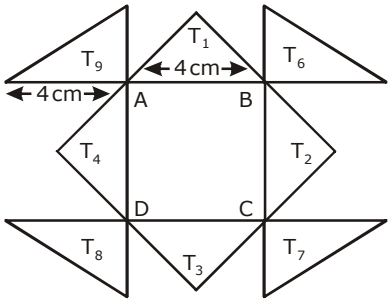
(A) 2 (B) 4 (C) 3 (D) $\sqrt{2}$

Q.41 The perimeter of a rhombus is 160 cm and one diagonal is 10 cm long. The length of other diagonal is

(A) $50\sqrt{35}$ cm (B) $150\sqrt{7}$ cm

(C) $15\sqrt{41}$ cm (D) $150\sqrt{10}$ cm

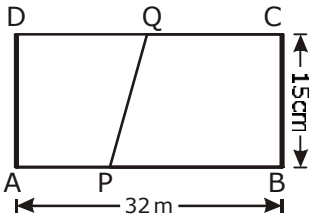
Q.42 In the given figure ABCD is a square of side 4 cm. T_1, T_2, T_3, T_4 are equilateral triangles and T_5, T_6, T_7, T_8 are isosceles right angled triangles with base as 4 cm. Total area of figure is



(A) $16(\sqrt{3}+1)$ cm^2 (B) $16\sqrt{3}\,(\sqrt{3}+1)$ cm^2

(C) $15(\sqrt{3}-1)$ cm^2 (D) $48(\sqrt{3}+1)$ cm^2

Q.43 ABCD is a rectangle. A point P on AB and another point Q on DC is taken such that PB : QC = 3 : 1. If area of shaded region of $\frac{3}{8}$ times the area of rectangle then lengths PB and QC respectively are



(A) 6 cm, 2 cm (B) 12 cm, 4 cm
(C) 24 cm, 8 cm (D) none of these

Q.44 The length of shortest altitude of triangle with sides 52 cm, 56 cm and 60 cm is

(A) 44.8 cm (B) 51.7 cm
(C) 48 cm (D) none of these

Q.45 A rhombus ABCD is such that AB = 10 cm and $\angle B = 120^\circ$. Then area of rhombus ABCD =

(A) 40 cm^2 (B) 400 cm^2
(C) 200 cm^2 (D) $50\sqrt{3}$ cm^2

Q.46 The area of a right angled triangle is 600 sq. cm. If base of the triangle exceeds the altitude by 10 cm, the dimensions of the triangle are

(A) 120, 100, 130 (B) 30, 40, 60
(C) 30, 40, 50 (D) none of these

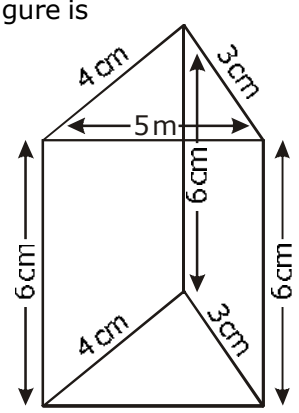
Q.47 The parallel sides AB and DC of a trapezium ABCD are 15 cm and 25 cm respectively. If each of non-parallel sides is 10 cm, the area of trapezium is

(A) 100 cm^2 (B) $100\sqrt{3}$
(C) 300 cm^2 (D) none of these

Q.48 A triangle has its sides as 13 cm, 14 cm and 15 cm. A parallelogram having double the area of this triangle stands on side 14 cm of this triangle. Then height of parallelogram is

(A) 12 cm (B) 10 cm
(C) 8 cm (D) none of these

Q.49 Total surface are of rectangular prism as shown in the figure is



(A) 20 cm^2 (B) 12 cm^2
(C) 96 cm^2 (D) none of these

Q.50 A diagonal of a square field is 40 m, then area of the field is

(A) 800 m^2 (B) 1600 m^2
(C) 400 m^2 (D) none of these

Q.51 If the area of an equilateral triangle is $16\sqrt{3}$ cm^2, then the perimeter of the triangle is

(A) 48 cm (B) 24 cm
(C) 12 cm (D) 306 cm

Q.52 The sides of a triangle are 35 cm, 54 cm and 61 cm respectively. The length of its longest altitude is

(A) $16\sqrt{5}$ cm (B) $10\sqrt{5}$ cm
(C) $24\sqrt{5}$ cm (D) 28 cm

Q.53 An isosceles right angled triangle has area 8 cm^2. The length of its hypotenuse is

(A) $\sqrt{32}$ cm (B) $\sqrt{16}$ cm
(C) $\sqrt{48}$ cm (D) $\sqrt{24}$ cm

Q.54 In a ΔABC it is given that base = 12 cm and height = 5 cm. Its area is

(A) 60 cm^2 (B) 30 cm^2
(C) $15\sqrt{3}$ cm^2 (D) 45 cm^2

Q.55 The length of three sides of a triangle are 20 cm, 16 cm and 12 cm. The area of the triangle is

(A) 96 cm^2 (B) 120 cm^2

(C) 144 cm^2 (D) 160 cm^2

Q.56 Each side of an equilateral triangle measures 8 cm. The area of the triangle is

(A) $8\sqrt{3}$ cm^2 (B) $16\sqrt{3}$ cm^2

(C) $32\sqrt{3}$ cm^2 (D) 48 cm^2

Q.57 The base of an isosceles triangle is 8 cm long and each of its equal sides measures 6 cm. The area of the triangle is

(A) $16\sqrt{5}$ cm^2 (B) $8\sqrt{5}$ cm^2

(C) $16\sqrt{3}$ cm^2 (D) $8\sqrt{3}$ cm^2

Q.58 The base of an isosceles triangle is 6 cm and each of its equal is 5 cm. The height of the triangle of the triangle is

(A) 8 cm (B) $\sqrt{30}$ cm

(C) 4 cm (D) $\sqrt{11}$ cm

Q.59 Each of the two equal sides of an isosceles right triangle is 10 cm long. Its area is

(A) $5\sqrt{10}$ cm^2 (B) 50 cm^2

(C) $10\sqrt{3}$ cm^2 (D) 75 cm^2

Q.60 Each side of an equilateral triangle is 10 cm long. The height of the triangle is

(A) $10\sqrt{3}$ cm (B) $5\sqrt{3}$ cm

(C) $10\sqrt{2}$ cm (D) 5 cm

ANSWER KEY

1.	A	**2.**	C	**3.**	B	**4.**	A
5.	B	**6.**	A	**7.**	C	**8.**	C
9.	A	**10.**	C	**11.**	A	**12.**	A
13.	A	**14.**	A	**15.**	B	**16.**	C
17.	C	**18.**	D	**19.**	B	**20.**	B
21.	A	**22.**	C	**23.**	A	**24.**	C
25.	A	**26.**	A	**27.**	B	**28.**	C
29.	A	**30.**	A	**31.**	B	**32.**	C
33.	A	**34.**	B	**35.**	D	**36.**	B
37.	C	**38.**	B	**39.**	A	**40.**	B
41.	B	**42.**	B	**43.**	D	**44.**	A
45.	D	**46.**	C	**47.**	B	**48.**	A
49.	D	**50.**	A	**51.**	C	**52.**	A
53.	A	**54.**	B	**55.**	A	**56.**	B
57.	B	**58.**	C	**59.**	B	**60.**	B

1. The base of a right angled triangle is 5 metres and hypotenuse is 13 metres. Its area will be :

(A) 25 m^2 (B) 28 m^2 (C) 30 m^2 (D) None of these

2. The sides of a triangular board are 13 metres, 14 metres and 15 metres. The cost of painting it at the rate of Rs. 8.75 per m^2 is

(A) Rs. 688.80 (B) Rs. 735 (C) Rs. 730.80 (D) Rs. 722.50

3. The area of an equilateral triangle whose side is 8 cm, is

(A) 64 cm^2 (B) $16\sqrt{3}$ cm^2 (C) 21.3 cm^2 (D) $4\sqrt{3}$ cm^2

4. The length of each side of an equilateral triangle having an area of $4\sqrt{3}$ cm^2, is :

(A) 4 cm (B) $\frac{4}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{4}$ (D) 3 cm

5. If x is the length of a median of an equilateral triangle, then its area is :

(A) x^2 (B) $\frac{x^2\sqrt{3}}{2}$ (C) $\frac{x^2\sqrt{3}}{3}$ (D) $\frac{x^2}{2}$

6. The altitude of an equilateral triangle of side $2\sqrt{3}$ cm is :

(A) $\frac{\sqrt{3}}{2}$ cm (B) $\frac{1}{2}$ cm (C) $\frac{\sqrt{3}}{4}$ cm (D) 3 cm

7. In a triangle ABC, BC = 5 cm, AC = 12 cm and AB = 13 cm. The length of the altitude drawn from B on AC is :

(A) 4 cm (B) 5 cm (C) 6 cm (D) 7 cm

8. A triangle of area 9 × y cm^2 has been drawn such that its area is equal to the area of an equilateral triangle of side 6 cm. Then, the value of y is

(A) $\sqrt{2}$ cm (B) $\sqrt{3}$ cm (C) 2 cm (D) 3 cm

9. In Δ PQR, side QR = 10 cm and height PM = 4.4 cm. If PR = 11 cm, then altitude QN equals :

(A) 4 cm (B) 5 cm (C) 5.5 cm (D) 5.6 cm

10. The area of a right angled triangle is 30 cm^2 and the length of its hypotenuse is 13 cm. The length of the shorter leg is:

(A) 4 cm (B) 5 cm (C) 6 cm (D) 7 cm

11. Area of a square with side x is equal to the area of a triangle with base x. The altitude of the triangle is :

(A) $\frac{x}{2}$ (B) x (C) 2x (D) 4x

12. A plot of land is in the shape of a right angled isosceles triangle. The length of the hypotenuse is $50\sqrt{2}$ m. The cost of fencing it at Rs. 3 per metre will be :

(A) less than Rs. 300 (B) less than Rs. 400 (C) more than Rs. 500 (D) more than Rs. 600

13. The perimeter of an isosceles triangle is equal to 14 cm, the lateral side is to the base in the ratio 5 : 4. The area of the triangle is

(A) $\frac{1}{2}\sqrt{21}$ cm^2 (B) $\frac{3}{2}\sqrt{21}$ cm^2 (C) $\sqrt{21}$ cm^2 (D) $2\sqrt{21}$ cm^2

14. If the area of an equilateral triangle is $24\sqrt{3}$ sq. m, then its perimeter is :

(A) 96 m (B) $12\sqrt{6}$ m (C) $4\sqrt{6}$ m (D) $2\sqrt{6}$ m

15. The ratio of the area of a square of side a and equilateral triangle of side a, is :

(A) 2 : I (B) 2 : $\sqrt{3}$ (C) 4 : 3 (D) 4 : $\sqrt{3}$

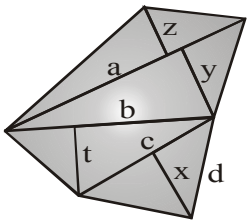
16. If every side of a trianlge is doubled, then increase in the area of the triangle is :

(A) $(\sqrt{2}\times 100)\%$ (B) 200 % (C) 300 % (D) 400 %

17. If the altitude of an equilateral triangle is $\sqrt{6}$, then its area is :

(A) $3\sqrt{3}$ (B) $2\sqrt{3}$ (C) $2\sqrt{2}$ (D) $6\sqrt{2}$

18. A surveyor in his field book has drawn the plot as shown in the given figure.
The area of the plot is :



(A) $\frac{1}{2}$ (az + by + ct + dx) (B) $\frac{1}{2}$ (bt + cx + ay + az)

(C) $\frac{1}{2}$ (cx + bt + by +az) (D) $\frac{1}{2}$ (d + t) (c + x) + $\frac{1}{2}$(a + b) (y + z)

19. If an equilateral triangle of area X and a square of area Y have the same perimeter, then :

(A) X > Y (B) X = Y (C) X < Y (D) X $\leq$ Y

20. A square and an equilateral triangle have equal perimeters. If the diagonal of the square is $12\sqrt{2}$ cm, then the area of the triangle is :

(A) $24\sqrt{2}$ cm^2 (B) $24\sqrt{3}$ cm^2 (C) $48\sqrt{3}$ cm^2 (D) $64\sqrt{3}$ cm^2

(OBJECTIVE)			ANSWER KEY						EXERCISE # 4	
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	C	B	B	A	C	D	B	B	A	B
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	C	C	D	B	D	C	B	B	C	D

SURFACE AREA AND VOLUME

INTRODUCTION

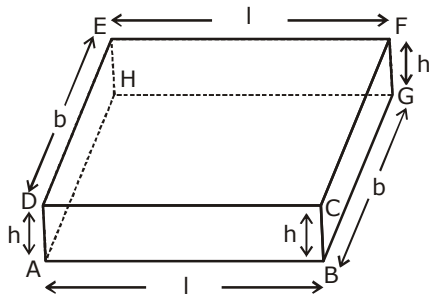
We are familiar with different kinds of plane figures for example, rectangle, square, circle etc. We can find area and perimeter of these plane figures.

If we cut out many pieces of plane figures of same shape and size and stack them up in a vertical pile, we obtain solid figures. For example, the stack of squares or rectangle give a cuboid, and the stack of circles give a circular cylinder.

In the present chapter, We shall study to find the surface areas and volumes of different solids such as cube, cuboid, cylinder, cone and sphere.

SURFACE AREA OF A CUBOID

A solid figure having length, breadth and height is called a cuboid. It has 6 rectangular faces, 12 edges and 8 vertices. For example, a match box, a brick an almirah are all in the shape of a cuboid.



Here ABCDEFGH is a cuboid having 6 rectangular faces ABCD, EFGH, BCFG, ADEH, ABGH and CDEF; 12 edges AB, DC,EF, HG, AH, DE, CF, BG, AD, HE, GF and BC; 8 vertices A,B,C,D,E,F,G and H.

Surface area of cuboid is the sum of the areas of its 6 rectangular faces.

Area of face ABCD = l × h = lh

Area of face EFGH = l × h = lh

Area of face BCFG = b × h = bh

Area of face ADEH = b× h = bh

Area of face ABGH = l × b = /b

Area of face CDEF = l × b = /b

∴ Total surface area of cuboid = 2 (lb + bh + hl).

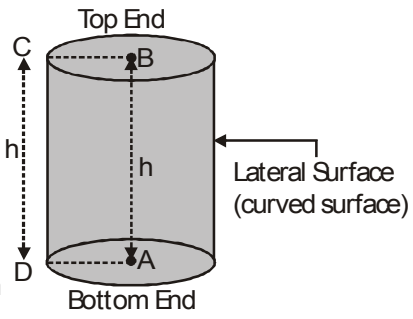
Also the area of four walls of cuboid = lateral surface area of cuboid = 2 (bh + hl) = 2(l + b)×h.

Surface area of a cube. We know that if length, breadth and height of a cuboid are equal it becomes a cube. So surface area of a cube can be found by taking l = b = h.

∴ Total surface area of cube = $6a^2$ and lateral surface area of cube (area of four walls) = $4a^2$.

(Note. When simply surface area is written, it means total area).

RIGHT CIRCULAR CYLINDER



Solids like circular pillars, circular pipes, circular pencils, measuring jars, road rollers and gas cylinders, etc., are said to be in cylindrical shape.

In mathematical terms, ***a right circular cylinder is a solid generated by the revolution of a rectangle about its sides.***

Let the rectangle ABCD revolve about its side AB, so as to describe a right circular cylinder as shown in the figure.

You must have observed that the cross-sections of a right circular cylinder are circles congruent and parallel to each other.

FORMULAE

Area of each circular end $= \pi r^2$ sq. units.

Curved (Lateral) Surface Area $= (2\pi rh)$ sq. units.

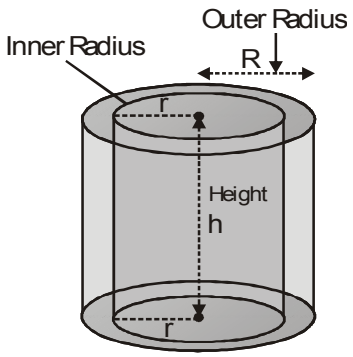
Total Surface Area = Curved Surface Area + Area of two circular ends.
$= (2\pi rh + 2\pi r^2)$ sq. units.
$= [2\pi r (h + r)]$ sq. units.

Volume of cylinder $= \pi r^2 h$ cubic units.

HOLLOW RIGHT CIRCULAR CYLINDERS

Solids like iron pipes, rubber tubes, etc., are in the shape of hollow cylinders.

A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder.



For a hollow cylinder of height h and with external and internal radii R and r respectively, we have :

FORMULAE

Thickness of cylinder = (R – r) units.

Area of a cross-section = $(\pi R^2 - \pi r^2)$ sq. units.
$= \pi (R^2 - r^2)$ sq. units

Curved (Lateral) Surface Area = (External Curved Surface Area) + (Internal Curved Surface Area)
= $(2\pi Rh + 2\pi rh)$ sq. units = $2\pi h (R + r)$ sq. units.

Total Surface Area = (Curved Surface Area) + 2 (Area of Base Ring)
= $[(2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)]$ sq. units
= $2\pi(Rh + rh + R^2 - r^2)$ sq. units.

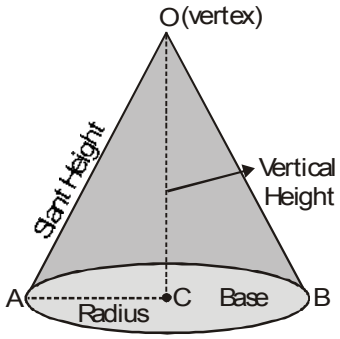
Volume of Material = $\pi(R^2 - r^2)h$ cubic units

Volume of Hollow region = $\pi r^2 h$ cubic units

RIGHT CIRCULAR CONE

Solids like an ice-cream cone, a conical tent, a conical vessel, a clown's cap etc. are said to be in conical shape. In mathematical terms, ***a right circular cone is a solid generated by revolving a right-angled triangle about one of the sides containing the right angle.***

Let a triangle AOC revolve about it's side OC, so as to describe a right circular cone, as shown in the figure.



FORMULAE

Area of the curved (lateral) surface = $\pi r\ell$ sq. units

Total Surface Area of cone = (Curved surface Area + Area of Base)

$= \pi r\ (\ell + r)$ sq. units

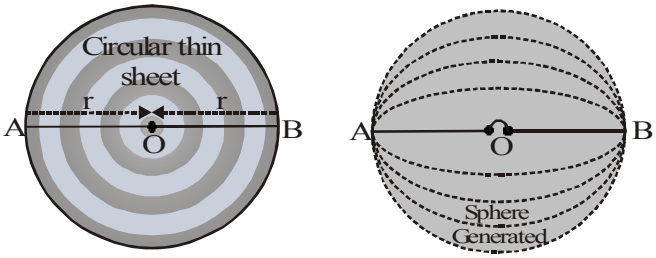
Volume of cone $= \frac{1}{3}\pi r^2 h$ cubic units.

SPHERE

Objects like football, volleyball, throw-ball etc. are said to have the shape of a sphere.

In mathematical terms, ***a sphere is a solid generated by revolving a circle about any of its diameters.***

Let a thin circular disc of card board with centre O and radius r revolve about its diameter AOB to describe a sphere as shown in figure.



Here, O is called the **centre of the sphere** and r is **radius of the sphere**. Also, the line segment AB is a **diameter of the sphere**.

For a solid sphere of radius = r, we have :

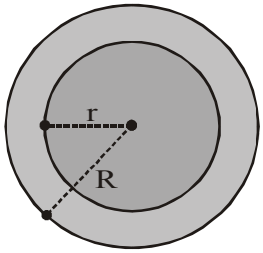
FORMULAE

Surface area of the sphere $= (4\pi r^2)$ sq. units.

Volume of the sphere $= \left(\frac{4}{3}\pi r^3\right)$ cubic units.

SPHERICAL SHELL

The solid enclosed between two concentric spheres is called a spherical shell.



For a spherical shell with external radius = R and internal radius = r, we have :

FORMULAE

Thickness of shell = (R – r) units.

Outer surface area = $4\pi R^2$ sq. units.

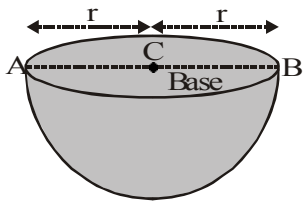
Inner surface area = $4\pi r^2$ sq. units.

Volume of material = $4/3\ \pi(R^3 - r^3)$ cubic units.

HEMISPHERE

When a plane through the centre of a sphere cuts it into two equal parts, then each part is called a hemisphere.

From a solid sphere, the obtained hemisphere is also a solid and it has a base as shown in fig.



For a hemisphere of radius r, we have :

FORMULAE

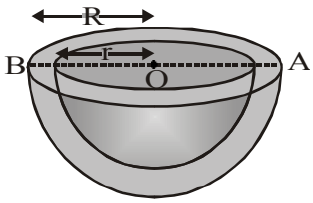
Curved surface area = $2\pi r^2$ sq. units.

Total Surface Area = $(2\pi r^2 + \pi r^2) = 3\pi r^2$ sq. units.

Volume = $\frac{2}{3}\pi r^3$ cubic units.

HEMISPHERICAL SHELL

The solid enclosed between two concentric hemispheres is called a hemispherical shell.



For a hemispherical shell of external radius = R and internal radius = r, we have :

FORMULAE

Thickness of the shell = (R – r) units.

Outer curved surface area = $(2\pi R^2)$ sq. units.

Inner curved surface area = $(2\pi r^2)$ sq. units.

Total surface area = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2) = \pi(3R^2 + r^2)$ sq. units.

Volume of the material = $\frac{2}{3}\pi(R^3 - r^3)$ cubic units.

SOLVED PROBLEMS

Ex.1 Hameed has built a cubical water tank with lid for his house, with each other edge 1.5m long. He gets the outer surface of the tanks excluding the base, covered with square tiles of side 25 cm. Find how much he would spend for the tiles, if the cost of the tiles is Rs. 360 per dozen.

Sol. The tank edge (A) = 1.5m = 150 cm.

$\therefore$ Surface area of tank excluding the base = $5a^2 = 5\times150\times150 cm^2$.

Also area of 1 tile = $25\times25 cm^2$

$$\therefore \text{Number of tiles required} = \frac{\text{Surface area of tank excluding the base}}{\text{Area of 1 tile}}$$

$$= \frac{5\times150\times150}{25\times25} = 180 \text{ tiles}$$

$$\text{Number of dozens of tiles required} = \frac{180}{12} = 15 \text{ dozen}$$

Cost of 1 dozen of tiles = Rs. 360

$\therefore$ cost of 15 dozen of tiles = Rs (15×360) = Rs. 5400.

Ex.2 Shanti sweets stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The Bigger of dimensions 25cm × 20 cm × 5cm. and the smaller of dimensions 15 cm × 12cm × 5cm. For all the overlaps 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 per 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind. **[NCERT]**

Sol. For bigger box,

l = 25cm, b= 20 cm, h = 5cm

$\therefore$ Surface area of the bigger box

= 2 (lb + bh + hl)

= 2 (25 × 20 + 20 × 5+5×25)

= 2 (500 + 100 +125)

= 2 ×725 = 1450 cm^2

For smaller box,

ℓ = 15cm b = 12cm, h = 5cm

$\therefore$ Surface area of the smaller box = 2 (lb + bh + hl)

= 2(15×12+12×5+5×15)

= 2 (180 + 60 + 75)

= 2 × 315 = 630 cm^2

$\therefore$ Total surface area of two boxes

= (1450 + 630)cm^2 = 2080 cm^2.

$\therefore$ Extra area of cardboard required for overlapping = 5 per cent of area of boxes

$$= \frac{5}{100}\times2080 = 104 cm^2.$$

$\therefore$ Area of cardboard needed to make 1 box of each kind = 2080 + 104 = 2184 cm^2.

$\therefore$ Total area of cardboard required to make 250 boxes of each kind = 250 × 2184 cm^2.

Rate of 1000 cm^2 of cardboard = Rs. 4

$$\therefore \text{Cost of total cardboard} = \text{Rs. } \frac{250\times2184\times4}{1000} = \text{Rs. } 2184.$$

Ex.3 The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs. 15000, find the height of the hall. **[NCERT]**

Sol. [Hint : Let the height of the hall be h m.

Area of 4 walls = $2(\ell + b)\, h$ = perimeter × h

Then, $250 \times h \times 10 = 15000$

$\Rightarrow$ h = 6m.

Ex.4 Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with front face as a flab which can be rolled up). Assuming that the stitching margins are very small and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m , with base dimensions 4m × 3m? **[NCERT]**

Sol. For the box like structure,

l = 4m, b= 3m, h = 2.5m.

$\therefore$ Surface area of the shelter = lb + 2 (bh + hl)

= 4×3+2 (3×2.5+2.5×4)

= 12 + 2 (7.5 + 10)

= 12 +35 = 47 m^2.

$\therefore$ Area of tarpaulin required = 47m^2.

Ex.5 The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container? **[NCERT]**

Sol. For a brick

length, ℓ = 22.5 cm, breadth, b = 10 cm, height, h = 7.5 cm

$\therefore$ Total surface area of a brick = $2(\ell b + bh + h\ell)$

= $2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)$

= $2(225 + 75 + 168.75)$

= $2(468.75) = 937.5\ cm^2 = .09375\ m^2$

$\therefore$ Number of bricks that can be painted out = $\frac{9.375}{.09375} = 100$

Hence, 100 bricks can be pained out of the given container.

Ex.6 A small indoor greenhouse is made entirely of glass sheets (including the base) held together with tape. It is 40 cm long, 30 cm wide and 30 cm high. Find **[NCERT]**

(i) the area of the glass sheet required and

(ii) the total length of the tape required for all the 12 edges.

Sol. The dimensions of the greenhouse are as under :

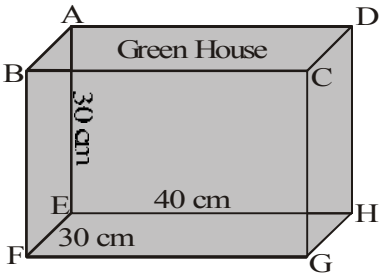
Length (ℓ) = 40 cm, Width (b) = 30 cm, Height (h) = 30 cm

(i) The area of the glass sheet required

= The total outer (or inner) surface area of the greenhouse

= $2\ [\ell \times b + b \times h + h \times \ell]$

= $2\ [40 \times 30 + 30 \times 30 + 30 \times 40]\ cm^2$



= 2 [1200 + 900 + 1200] cm^2

= 2 × 3300 cm^2 = 6600 cm^2

Hence, 6600 cm^2 of glass sheet is required.

(ii) Length of the tap required = Sum of the length of the 12 edges.

= 4 (ℓ + b + h)

= 4 × [40 + 30 + 30] cm = 400 cm

Hence, 400 cm of the tape is required.

Ex.7 A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes? **[NCERT]**

Sol. Volume of a matchbox = 4 × 2.5 × 1.5 cm^3 = 15 cm^3

∴ Volume of a packet containing 12 such boxes = 15 × 12 cm^3 = 180 cm^3.

Ex.8 A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks of dimensions 24 cm × 12 cm × 8 cm, then find the number of bricks which are required.

Sol. We know that, the volume of the wall and the sum of the volumes of the required number of bricks is same.

Length of the wall = 10 × 100 cm = 1000 cm

Breadth or the thickness of the wall = 24 cm

Height of the wall = 4 × 100 cm = 400 cm

The wall is in the shape of a cuboid and its volume = 1000 × 24 × 400 cm^3

Now, a brick is also a cuboid having length = 24 cm, breadth = 12 cm and height = 8 cm.

Volume of one brick = 24 × 12 × 8 cm^3

The required number of bricks = $\frac{Volume\ of\ the\ wall}{Volume\ of\ one\ brick} = \frac{1000 \times 24 \times 400}{24 \times 12 \times 8} = \frac{5000}{12}$ = 4166.6

Hence, the required number of bricks = 4167.

Ex.9 The students of a vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder of base radius 3 cm and height 10.5 cm, using cardboard. The Vidyalaya was to supply the competitor with cardboard. if there were 35 competitors, how much cardboard was required to be bought for the competition ? **[NCERT]**

Sol. For one pen holder, r = 3 cm, h = 10.5 cm

Surface area of cardboard needed for 1 pen hodler

$= 2\pi rh + \pi r^2 = \pi r(2h + r)$

$= \frac{22}{7} \times 3(2 \times 10.5 + 3)$

$= \frac{22}{7} \times 3 \times 24 cm^2$

Surface area of cardboard needed for 35 pen holders

$= \frac{22}{7} \times 3 \times 24 \times 35 = 7920 cm^2$

Ex.10 In a hot water heating system, there is a cylindrical pipe of length 28 m,and diameter 5 cm. Find the total radiating surface in the system.

Sol. Here the length, h of the cylindrical pipe = 28 m and radius, r = $\frac{5}{2}$ cm = $\frac{5}{2 \times 100}$ m = $\frac{5}{200}$ m = $\frac{1}{40}$ m

∴ Total radiating surface in the system = $2\pi rh = 2 \times \frac{22}{7} \times \frac{1}{40} \times 28 = 4.4\ m^2$.

Ex.11 The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder. **[NCERT]**

Sol. Let the radius of the base of the cylinder be r cm.

height, h = 14 cm

Curved surface area = 88 cm^2

$\Rightarrow \quad 2\pi rh = 88$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 14 = 88$

$\Rightarrow \quad r = \frac{88 \times 7}{2 \times 22 \times 14}$

$\Rightarrow \quad r = 1 \quad \Rightarrow \quad 2r = 2$

Hence, the diameter of the base of the cylinder is 2 cm.

Ex.12 The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2. **[NCERT]**

Sol. Radius of the roller (r) = $\frac{84}{2}$ cm = 42 cm

length of the roller (h) = 120 cm

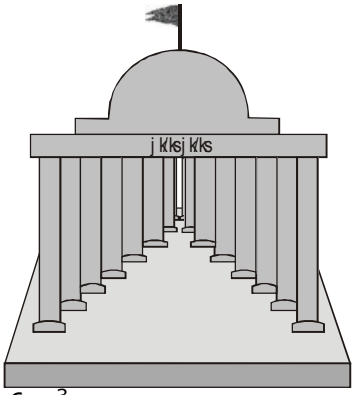
$\therefore$ Area of the playground levelled in taking 1 complete revolution = $2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120 = 31680\ cm^2$

$\therefore$ Area of the playground = 31680 × 500 = 15840000 $cm^2 = \frac{15840000}{100 \times 100} m^2 = 1584\ m^2$.

Hence, the area of the playground is 1584 m^2.

Ex.13 The pillars of a temple are cylindrical shaped. If each pillar has a circular base of radius 20 cm and height 7 m, then find the quantity of concrete mixture used to build 20 such pillars. Also find the cost of the concrete mixture at the rate of Rs. 200 per m^3. $\left(\text{Take } \pi = \frac{22}{7}\right)$ **[NCERT]**

Sol. Concrete mixture used for making each pillar = Volume of each pillar = $\pi r^2 \times h$



Radius of base of pillar, r = 20 cm = $\frac{20}{100}$ m = $\frac{1}{5}$ m

Height of pillar, h = 7m

$\therefore$ Volume of mixture used for 1 pillar

$= \frac{22}{7} \times \left(\frac{1}{5}\right)^2 \times 7\ m^3 = \frac{22}{25} m^3$

$\therefore$ Volume of mixture used for 20 pillars

$= 20 \times \frac{22}{25} m^3 = \frac{88}{5} m^3 = 17.6\ m^3$

Hence, volume of mixture required to make 20 pillars = 17.6 m^3

Now, cost of mixture at the rate of Rs.200 per m^3

= Rs.200 × 17.6 = Rs. 3520.

Hence, the cost of concrete mixture is Rs. 3520.

Ex.14 Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Sol. $\because$ Diameter of the base = 10.5 cm **[NCERT]**

$\therefore$ Radius of the base (r) = $\frac{10.5}{2}$ cm = 5.25 cm

Slant height (ℓ) = 10 cm

$\therefore$ Curved surface area of the cone = $\pi r\ell = \frac{22}{7} \times 5.25 \times 10 = 165\ cm^2$.

Ex.15 The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? (1000 cm^3 = 1ℓ)

Sol. Let the base radius of the cylindrical vessel be r cm.

Then, circumference of the base of the cylindrical vessel = $2\pi r$ cm.

According to the question, $2\pi r = 132$

$\Rightarrow 2 \times \frac{22}{7} \times r = 132 \Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21$ cm

Height of the cylindrical vessel, h = 25 cm

$\therefore$ Capacity of the cylindrical vessel = $\pi r^2 h = \frac{22}{7} (21)^2 (25)\ cm^3$

$= 34650\ cm^3 = \frac{34650}{1000}\ell = 34.65\ \ell$

Hence, the cylindrical vessel can hold 34.65 ℓ of water.

Ex.16 The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.

Sol. $\because$ Inner diameter = 24 cm

$\therefore$ Inner radius (r) = $\frac{24}{2}$ cm = 12 cm

$\because$ Outer diameter = 28 cm

$\therefore$ Outer radius (R) = $\frac{28}{2}$ cm = 14 cm

Length of the pipe (h) = 35 cm

Outer volume = $\pi R^2 h = \frac{22}{7} \times (14)^2 \times 35 = 21560\ cm^3$

Inner volume = $\pi r^2 h = \frac{22}{7} \times (12)^2 \times 35 = 15840\ cm^3$

$\therefore$ Volume of the wood used = Outer volume – Inner volume = 21560 cm^3 – 15840 cm^3 = 5720 cm^3

$\therefore$ Mass of the pipe = 5720×0.6 g = 3432 g = 3.432 kg

Hence, the mass of the pipe is 3.432 kg.

Ex.17 The ratio between the curved surface area and the total surface area of a right circular cylinder is 1:2. Find the ratio between the height and radius of the cylinder.

Sol. Let r be the radius and h be the height of the cylinder. Then according to question,

Curved surface area : total surface area = 1 : 2.

$$\Rightarrow \frac{2\pi h}{2\pi rh + 2\pi r^2} = \frac{1}{2} \Rightarrow \frac{2\pi rh}{2\pi r(r+h)} - \frac{1}{2}$$

$$\Rightarrow \frac{h}{r+h} = \frac{1}{2} \Rightarrow 2h = r + h \Rightarrow h = r$$

$\Rightarrow$ h : r = 1 : 1

Hence the required ratio is 1 : 1.

Ex.18 Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Sol. (i) Slant height (ℓ) = 14 cm

Curved surface area = 308 cm^2

$\Rightarrow \pi r\ell = 308 \Rightarrow \frac{22}{7} \times r \times 14 = 308$

$\Rightarrow r = \frac{308 \times 7}{22 \times 14} \Rightarrow r = 7$ cm

Hence, the radius of the base is 7 cm.

(ii) Total surface area of the cone = $\pi r(\ell + r) = \frac{22}{7} \times 7 \times (14 + 7) = \frac{22}{7} \times 7 \times 21 = 462\ cm^2$

Hence, the total surface area of the cone is 462 cm^2.

Ex.19 A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones ? (use π = 3.14 and take $\sqrt{1.04} - 1.02$) **[NCERT]**

Sol. For one cone, we have diameter of base = 40 cm = 0.40 m

$\therefore$ radius of base = $\frac{0.40m}{2}$ = 0.20 m

Also height of cone = 1m

Let l be the slant height of cone then

$l^2 = r^2 + h^2$

$\Rightarrow$ $l = \sqrt{(0.20)^2 + 1^2} = \sqrt{1.04} = 1.02m$

As cone is hollow and its outerside is to be painted, the area of 1 cone to be painted

= curved surface area of cone

= π r l

= 3.14 ×0.2 × 1.02 m^2

$\therefore$ Area of 50 such cones to be painted

= 50× 3.14 ×0.2 ×1.02

= 32.028 m^2

Also cost of painting 1m^2 area = Rs. 12

$\therefore$ Cost of painting 50 cones = Rs. 12× 32.028

= Rs. 384.336 = Rs. 384.34

Ex.20 A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres? **[NCERT]**

Sol. For conical pit

Diameter = 3.5 m

$\therefore$ Radius (r) = $\frac{3.5}{2}$ m = 1.75 m

Depth (h) = 12 m

$\therefore$ Capacity of the conical pit = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12\ m^3$

= 38.5 m^3 = 38.5 × 1000 ℓ

= 38.5 kl.

Ex.21 A corn cob (as shown in the fig.), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob ? **[NCERT]**

Sol. Since the grains of corn are found on the curved surface of the corn cob, so in order to find total number of grains on the corn cob, first we need to find the curved surface area of the corn cob.

For the corn cob which is shaped like a cone, we have

$r = 2.1$ cm and $h = 20$ cm

Let l be the slant height of cone, then

$$l = \sqrt{r^2 + h^2}$$

$$\Rightarrow \quad l = \sqrt{(2.1)^2 + (20)^2} = \sqrt{4.41 + 400}$$

$$= \sqrt{404.41}$$

$$= 20.11 \text{ cm}$$

$\therefore$ Curved surface area of corn cob $= \pi rl$

$$= \frac{22}{7} \times 2.1 \times 20.11$$

$$= 132.726 \text{ cm}^2$$

$$= 132.73 \text{ cm}^2$$

$\therefore$ Now. no. of grains on 1 cm^2 area = 4

$\therefore$ Total no. of grains on 132.73 cm^2 area = 132.73 ×4 = 530.92 = 531.

So, there would be approximately 531 grains of corn on the cob.

Ex.22 A river 3m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute? **[NCERT]**

Sol. Depth of river = 3m

Breadth of river = 40 m

Rate of flow of water = 2km/hr = $\frac{2000}{60}$ m per minute

i.e., In one minute length of river covered by water = $\frac{2000}{60}$ m

Hence volume of water coming in 1 minute = $\left(\frac{2000}{60} \times 40 \times 3\right) \text{m}^3 = 4000 \text{ m}^3$.

Ex.23 If V and S represent respectively the volume and surface area of a cuboid of length l, breadth b and height h. Prove that $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{l} + \frac{1}{b} + \frac{1}{h}\right)$

Sol. For cuboid of length l, breadth b and height h

$S = 2(lb + bh + hl)$ and $V = lbh$

$$\therefore \text{RHS} = \frac{2}{S}\left(\frac{1}{l} + \frac{1}{b} + \frac{1}{h}\right)$$

$$= \frac{2}{2(lb + bh + hl)}\left(\frac{bh + lh + lb}{lbh}\right) = \frac{1}{lbh}$$

$$= \frac{1}{V} = \text{LHS} \quad \text{Hence Proved.}$$

Ex.24 Water in a canal, of width 3m and depth 1.2m, is flowing with a velocity of 20 km per hour. How much area in hectare will it irrigate in 30 minutes if 9cm of standing water is desired. (use 10000 m^2 = 1hectares).

Sol. Width of canal = 3m Depth of canal = 1.2 m

In 1 hour the length of canal covered by flowing water = 20 km.

$\therefore$ In $\frac{1}{2}$hr = 30 minutes the length of canal covered by flowing water = 10 km = 10000m.

So volume of water coming through canal in 30 minutes = (10,000 × 3×1.2)m^3.

Also if this water is irrigating an area of A m^2 where height of standing water = 9cm = 0.09m. Then we should have

Volume of standing water = Volume of water coming through canal in 30 minutes

$\Rightarrow$ (A × 0.09) = (10,000 ×3 ×1.2)

$\Rightarrow$ $A = \frac{10,000 \times 3 \times 1.2}{0.09} m^2$

$\Rightarrow$ = 4,00,000 $m^2 = \frac{4,00,000}{10,000}$ hectares

= 40 hectares.

Ex.25 Find the capacity in litres of a conical vessel with (i) radius 7 cm, slant height 25 cm. (ii) height 12 cm, slant height 13 cm.

Sol. (i) r = 7 cm, ℓ = 25 cm

$r^2 + h^2 = \ell^2$

$\Rightarrow$ $(7)^2 + h^2 = (25)^2$

$\Rightarrow$ $h^2 = (25)^2 - (7)^2$

$\Rightarrow$ $h^2 = 625 - 49$

$\Rightarrow$ $h^2 = 576$

$\Rightarrow$ $h = \sqrt{576}$

$\Rightarrow$ h = 24 cm

$\therefore$ Capacity = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24 = 1232\ cm^3 = 1.232\ \ell$.

(ii) h = 12 cm, ℓ = 13 cm

$r^2 + h^2 = \ell^2$

$\Rightarrow$ $r^2 + (12)^2 = (13)^2$

$\Rightarrow$ $r^2 + 144 = 169$

$\Rightarrow$ $r^2 = 169 - 144$

$\Rightarrow$ $r^2 = 25$

$\Rightarrow$ $r = \sqrt{25}$

$\Rightarrow$ r = 5 cm

$\therefore$ Capacity = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12$

$= \frac{2200}{7}\ cm^3 = \frac{2200}{7000}\ \ell = \frac{11}{35}\ell$.

Ex.26 The diameter of the moon is approximately one-fourth the diameter of the earth. What fraction of the volume of the earth is the volume of the moon? **[NCERT]**

Sol. [Hint : Let d_1 and d_2 be the diameters of the moon and the earth respectively. Then,

$d_1 = \frac{1}{4} d_2$

$\Rightarrow \frac{r_1}{r_2} = \frac{1}{4}$; $\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{64}$

EXERCISE – I UNSOLVED QUESTIONS

Q.1 Find the surface area of a cube whose edge is 15 cm.

Q.2 A child playing with plastic building blocks which are of identical cubical shapes. She makes a structure as shown in figure. If the edge of each cube is 5 cm, then find the volume of the structure built by Aakriti.

Q.3 A cylindrical vessel, without lid, has to be tin-coated including both of its sides. If the radius of its base is $\frac{1}{2}$ m and its height is 1.4 m, calculate the cost of tin-coating at the rate of Rs. 50 per 1000 cm^2. (Use $\pi = 3.14$)

Q.4 A solid cylinder has total surface area of 462 cm^2. Its curved surface area is one third of its total surface area. Find the radius and height of the cylinder.

Q.5 The radii of two right circular cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 4. Calculate the ratio of their curved surface areas.

Q.6 The radius of the base of a conical tent is 12 m. The tent is 9 m high. Find the cost of the canvas required to make the tent, if one square metre of canvas costs Rs. 120. (Take $\pi = 3.14$)

Q.7 How many metres of cloth of 1.1 m width will be required to make a conical tent whose vertical height is 12 m and base radius is 16 m? Find also the cost of the cloth used at the rate of Rs.14 per metre.

Q.8 The curved surface area of a right circular cone is 198 cm^2 and the radius of its base is 7 cm. Find the volume of the cone. $\left(\text{Take } \pi = \frac{22}{7} \text{ and } \sqrt{2} = 1.41\right)$

Q.9 A hemispherical bowl is made from a metal sheet having thickness 0.3 cm. The inner radius of the bowl is 24.7 cm. Find the cost of polishing its outer surface at the rate of Rs. 4 per 100 cm^2. (Take $\pi = 3.14$)

Q.10 Find the amount of water displaced by a solid spherical ball of diameter 28 cm.

Q.11 There are 42 hemispherical bowls, each of radius 3.5 cm. Find the quantity of water in litres which is just sufficient to fill these 42 bowls. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Q.12 A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Q.13 Find the volume of a sphere whose surface area is 55.44 cm^2. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Q.14 The length of a hall is 20 m and breadth 16 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall.

Q.15 The sum of the length, breadth and height of a cuboid is 21 cm and the length of its diagonal is 13 cm. Find the surface area of the cuboid. Also find the cost of painting the surface at the rate of Rs. 1.40 per cm^2.

Q.16 The cost of papering the four walls of a room at 70 paise per square metre is Rs. 157.50. The height of the room is 5 metres. Find the length and the breadth of the room if they are in the ratio 4 : 1.

Q.17 A rectangular water reservoir is 10.8 m by 3.75 m at the base. Water flows into it at the rate of 18 m per second through a pipe having cross section 7.5 cm × 4.5 cm. Find the height to which the level of water reach in 15 minutes.

Q.18 The areas of three adjacent faces of a cuboid are p,q and r. If its volume is v, prove that $v^2 = pqr$.

Q.19 If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original one?

Q.20 Water is supplied to a city population for general use (not for drinking) from a river through a cylindrical pipe. The radius of the cross-section of the pipe is 20 cm. The speed of water through the pipe is 18 km per hour. Find the quantity of water in litres which is supplied to the city in two hours. (Take $\pi = 3.14$ and 1 m^3 = 1000 litres.)

Q.21 The base radii of the two right circular cones of the same height are in the ratio 3 : 5. Find the ratio of their volumes.

Q.22 If h, c and v be the height, curved surface and volume of a cone, show that $3\pi vh^3 - c^2h^2 + 9v^2 = 0$.

Q.23 A shot-put is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm^3, find the mass of the shot-put.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

Q.24 If the number of square centimetres on the surface of a sphere is equal to the number of cubic centimetres in its volume, what is the diameter of the sphere?

Q.25 A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

ANSWER KEY

1. 1350 cm^2 **2.** 1250 cm^3.

3. Rs.5181. **4.** r = 7cm, h = 3.5cm.

5. 5 : 6. **6.** Rs. 67824

7. $\frac{6400}{7}$ m, Rs. 12800.

8. 289.52 cm^3. **9.** Rs.157.

10. $11498\frac{2}{3}$ cm^3. **11.** 3.773 litres.

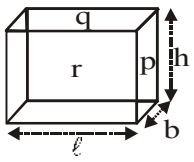
12. 0.06348 m^3 (Approx).

13. 38.808 cm^3. **14.** 8.88 m (approx).

15. 272 cm^2 ; Rs.380.80

16. 18 m ; 4.5 m **17.** 1.35 m.

18. [Hint :



$p = b \times h$; $q = \ell \times b$; $r = \ell \times h \Rightarrow pqr = \ell^2b^2h^2$]

19. 1 : 4. **20.** 4521600 litres.

21. 9 : 25 **23.** 3.845 kg (approx.)

24. 6 cm **25.** 2 : 1

EXERCISE – II SCHOOL EXAM/BOARD

Q.1 Write the lateral surface area of a cuboid having length ℓ units, breadth b units and height h. units.

Q.2 Write the total surface area of a cuboid having three edges of length as 10 cm, 5 cm and 3 cm.

Q.3 Write the curved surface area of a right circular cylinder whose radius is 3 cm and height is 5 cm.

Q.4 The volume of a right cylinder having base radius 10 cm is 600 π cm^3. Find the height of the cylinder.

Q.5 Write the curved surface area of a right circular cone having radius 7 cm and slant height 10 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Q.6 Write the total surface area of a right circular solid cone having radius 10 cm and slant height 25 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Q.7 Find the vertical height of a right circular cone whose radius is 6 cm and slant height is 10 cm.

Q.8 Find the volume of a right circular cylinder having radius 8 cm and height 10·5 cm. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Q.9 Write the volume of a right circular cone having radius r and height h.

Q.10 Find the quantity of water in litres in a hemispherical bowl of radius 21 cm. The bowl is completely filled with water. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Q.11 The volume of a cuboid is 440 cm^3 and the area of its base is 88 cm^2. Find its height.

Q.12 The volume of a cube is 1000 cm^3. Find its total surface area.

Q.13 How many 3 metre cubes can be cut from a cuboid measuring 18 m × 12 m × 9 m?

Q.14 The diameter of a right circular cone is 8 cm and its volume is 48 πcm^3. What is its height?

Q.15 A right circular cone is 3.6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Find its height.

Q.16 A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the water rises.

Q.17 A cone and a cylinder are having the same base. Find the ratio of their heights if their volumes are equal.

Q.18 Find the surface area and total surface area of a hemisphere of radius 21 cm.

Q.19 A sphere, a cylinder and a cone are of the same radius and same height. Find the ratio of their curved surface area.

Q.20 Show that the surface area of a sphere is the same as that of the lateral surface of a right circular cylinder that just encloses the sphere.

Q.21 The internal and external diameters of a hollow hemi-spherical vessel are 24 cm and 25 cm respectively. The cost of paint one sq. cm of the surface is 7 paise. Find the total cost to paint the vessel all over. (ignore the area of edge).

Q.22 Find the volume of a sphere whose surface area is 154 square cm.

Q.23 A solid sphere of radius 3 cm is melted and then cast into small spherical balls each of diameter 0.6 cm. Find the number of balls thus obtained.

Q.24 How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter.

Q.25 A solid lead ball of radius 7 cm was melted and then drawn into a wire of diameter 0.2 cm. Find the length of the wire.

Q.26 Length of a class-room is two times its height and breadth is $1\frac{1}{2}$ times its height. The cost of white-washing the walls at the rate of Rs. 1.60 per m^2 is Rs. 179.20. Find the cost of tiling the floor at the rate of Rs. 6.75 per m^2.

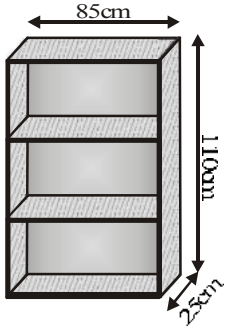
Q.27 The dimensions of a rectangular box are in the ratio 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rate of Rs. 4 and Rs. 4.50 per square metre is Rs. 416. Find the dimensions of the box.

Q.28 Find the number of bricks, each measuring 25 cm × 12.5 cm × 7.5 cm required to construct a wall 6 m long, 5 m high and 0.5 m thick, while the cement and sand mixture occupies 1/20 of the volume of the wall.

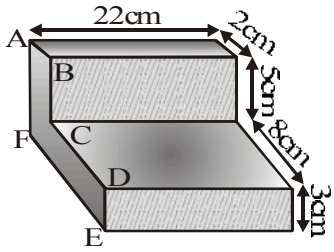
Q.29 A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m × 1.4 m and three windows, each measuring 2 m × 1m. The interior walls are to be colour washed. The contractor charges Rs. 5.25 per sq. m. Find the cost of colour washing.

Q.30 A room is half as long again as it is broad. The cost of carpeting the room at Rs. 3.25 per m^2 is Rs. 175.50 and the cost of papering the walls at Rs. 1.40 per m^2 is Rs. 240.80. If 1 door and 2 windows occupy 8 m^2, find the dimensions of the room.

Q.31 A wooden bookshelf has external dimensions as follows : Height = 110 cm, Depth = 25 cm, Breadth = 85 cm. The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2. Find the total expenses required for polishing and painting the surface of the bookshelf.



Q.32 In fig. the shape of a solid csopper piece (made of two pieces with dimensions as shown in the figure) is shown. The face ABCDEFA is the uniform cross section. Assume that the angle at A,B,C,D,E and F are right angles. Calculate the volume of the piece.



Q.33 A plot of land in the form of a rectangle has a dimension 240 m × 180 m. A drainlet 10 m wide is dug all around it (on the outside) and the earth dug out is evenly spread over the plot, increasing its surface level by 25 cm. Find the depth of the drainlet.

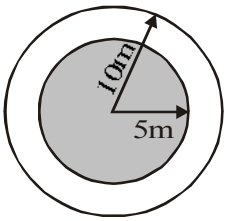
Q.34 A metallic sheet is of the rectangular shape with dimensions 48 cm × 36 cm. From each one of its corners, a square of 8 cm is cutoff. An open box is made of the remaining sheet. Find the volume of the box.

Q.35 Water in a canal, 30 dm wide and 12 dm deep, is flowing with a velocity of 20 km per hour. How much area will it irrigate in 30 min, if 9 cm of standing water is desired?

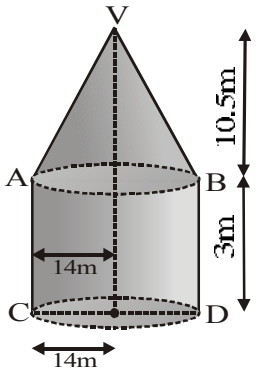
Q.36 A cylindrical road roller made of iron is 1 m wide. Its inner diameter is 54 cm and thickness of the iron sheet rolled into the road roller is 9 cm. Find the weight of the roller if 1 cubic cm of iron weighs 8 gm.

Q.37 A solid cylinder has total surface area of 462 square cm. Its curved surface area is one-third of its total surface area. Find the volume of the cylinder. (Take π = 22/7)

Q.38 A well with 10 m inside diameter is dug 14 m deep. Earth taken out of it is spread all around to a width of 5 m to form an embankment. Find the height of embankment.



Q.39 A tent is of the shape of a right circular cylinder upto a height of 3 metres and then becomes a right circular cone with a maximum height of 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs. 2 per square metre, if the radius of the base is 14 metres.

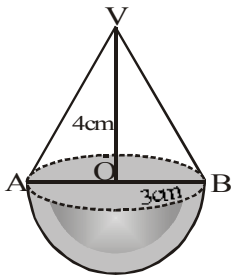


Q.40 A solid cube of side 7 cm is melted to make a cone of height 5 cm, find the radius of the base of the cone.

Q.41 From a right circular cylinder with height 10 cm and radius of base 6 cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid.

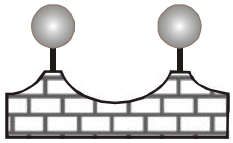
Q.42 The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost to paint 1 cm^2 surface is Rs. 0.05. Find the total cost to paint the vessel all over. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Q.43 A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 6 cm and its height is 4 cm. Find the cost of painting the toy at the rate of Rs. 5 per 1000 cm^2.



Q.44 The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in fig. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.

[NCERT]



Q.45 A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.

ANSWER KEY

1.	2 (ℓ + b)h	**2.**	190 cm^2
3.	30 πcm^2	**4.**	6 cm
5.	220 cm^2	**6.**	1100 cm^2
7.	8 cm	**8.**	2112 cm^3
9.	$\frac{1}{3}\pi r^2 h$	**10.**	19.404 litres
11.	5 cm	**12.**	600 cm^2
13.	72	**14.**	9 cm
15.	6.4 cm	**16.**	2 cm
17.	3 : 1		
18.	2772 cm^2 ; 4158 cm^2	**19.**	4 : 4 : $\sqrt{5}$
21.	Rs. 132.11	**22.**	179.66 cm^3
23.	1000	**24.**	2541
25.	457.33 m	**26.**	Rs. 324
27.	8 m, 12 m and 16 m	**28.**	6080
29.	Rs. 513.45		
30.	length = 9m, breadth = 6m and height = 6 m		
31.	Rs. 6275	**32.**	880 cm^3
33.	1.227 m	**34.**	5120 cm^3
35.	4,00,000 m^2	**36.**	1424.304 kg
37.	539 cm^3	**38.**	4.66 m
39.	Rs. 2068	**40.**	8.09 cm
41.	754.28 cm^3	**42.**	Rs. 96.28
43.	51 paise	**44.**	Rs. 382.80
45.	3 cm		

EXERCISE – III MULTIPLE CHOICE QUESTIONS

Q.1 The length, breadth and height of a cuboid are 15 cm, 12 cm and 4.5 cm respectively. Its volume is
(A) 243 cm^3 (B) 405 cm^3
(C) 810 cm^3 (D) 603 cm^3

Q.2 A cuboid is 12 cm long, 9 cm broad and 8 cm high. Its total surface area is
(A) 864 cm^2 (B) 552 cm^2
(C) 432 cm^2 (D) 276 cm^2

Q.3 The length, breadth and height of a cuboid are 15 m, 6 m and 5 dm respectively. The lateral suface area of the cuboid is
(A) 45 m^2 (B) 21 m^2
(C) 201 m^2 (D) 90 m^2

Q.4 A beam 9 m long, 40 cm wide and 20 cm high is made up of iron which weighs 50 kg per cubic metre. The weight of the beam is
(A) 27 kg (B) 48 kg
(C)36 kg (D) 56 kg

Q.5 The length of the longest rod that can be placed in a room of dimensions (10 m × 10m × 5m) is
(A) 15 m (B) 16 m
(C) $10\sqrt{5}$ m (D) 12 m

Q.6 What is the maximum length of a pencil that can be palced in a rectangular box of dimensions (8 cm × 6 cm × 5 cm) ? (Given $\sqrt{5} = 2.24$)
(A) 8 cm (B) 9.5 cm
(C) 19 cm (D) 11.2 cm

Q.7 The number of planks of dimensions (4 m × 5m ×2m) that can be stored in a pit which is 40 m long, 12 m wide and 16 cm deep is.
(A) 190 (B) 192
(C) 184 (D) 180

Q.8 How many planks of dimensions (5 m × 25cm × 10cm) can be stored in a pit which is 20 m long , 6 m wide and 50 cm deep ?
(A) 480 (B) 450
(C) 320 (D) 360

Q.9 How many bricks will be required to construct a wall 8 m long, 6 m high and 22.5 cm thick if each brick measures (25 cm × 11.25 cm × 6cm) ?
(A) 4800 (B) 5600
(C) 6400 (D) 5200

Q.10 How many persons can be accommodated in a dining hall of dimensions (20 m × 15 m × 4.5 m), assuming that each person requires 5 m^3 of air ?
(A) 250 (B) 270
(C) 320 (D) 300

Q.11 A river 1.5 m deep and 30 m wide is flowing at the rate of 3 km per hour. The volume of water that runs into the sea per minute is
(A) 2000 m^3 (B) 2250 m^3
(C) 2500 m^3 (D) 2750 m^3

Q.12 The lateral surface area of a cube is 256 m^2. The volume of the cube is
(A) 64 m^3 (B) 216 m^3
(C) 256 m^3 (D) 512 m^3

Q.13 The total surface area of a cube is 96 cm^2. The volume of the cube is
(A) 8 cm^3 (B) 27 cm^3
(C) 64 cm^3 (D) 512 cm^3

Q.14 The volume of a cube is 512 cm^3. Its total surface area is
(A) 256 cm^2 (B) 384 cm^2
(C) 512 cm^2 (D) 64 cm^2

Q.15 The length of the longest rod that can fit in a cubical vessel of side 10 cm, is
(A) 10 cm (B) 20 cm
(C) $10\sqrt{2}$ cm (D) $10\sqrt{3}$ cm

Q.16 If the length of diagonal of a cube is $8\sqrt{3}$ cm , then its surface area is
(A) 192 cm^2 (B) 384 cm^2
(C) 512 cm^2 (D) 768 cm^2

Q.17 If each edge of a cube is increased by 50%, then the percentage increase in its surface area is
(A) 50% (B) 75%
(C) 100% (D) 125%

Q.18 Three cubes of metal with edges 3 cm, 4 cm and 5 cm respectively are melted to form a single cube. The lateral surface area of the new cube formed is
(A) 72 cm^2 (B) 144 cm^2
(C) 128 cm^2 (D) 256 cm^2

Q.19 In a shower, 5 cm of rain falls. What is the volume of water that falls on 2 hectares of ground ?
(A) 500 m^3 (B) 750 m^3
(C) 800 m^3 (D) 1000 m^3

Q.20 Two cubes have their volumes in the ratio 1 : 27.The ratio of their surface areas is
(A) 1 : 3 (B) 1 : 8
(C) 1 : 9 (D) 1 : 18

Q.21 If each of a cube is doubled, then its volume
(A) is doubled (B) becomes 4 times
(C) becomes 6 times (D) becomes 8 times

Q.22 The diameter of the base of a cylinder is 6 cm and its height is 14 cm. The volume of the cylinder is
(A) 198 cm^3 (B) 396 cm^3
(C) 495 cm^3 (D) 297 cm^3

Q.23 If the diameter of a cylinder is 28 cm and its height is 20 cm, then its curved surface area is
(A) 880 cm^2 (B) 1760 cm^2
(C) 3520 cm^2 (D) 2640 cm^2

Q.24 If the curved surface area of a cylinder is 1760 cm^2 and its base radius is 14 cm, then its height is
(A) 10 cm (B) 15 cm
(C) 20 cm (D) 40 cm

Q.25 The height of a cylinder is 14 cm and its curved surface area is 264 cm^2. The volume of the cylinder is
(A) 308 cm^3 (B) 396 cm^3
(C) 1232 cm^3 (D) 1848 cm^3

Q.26 The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3. The height of the pillar is
(A) 4 m (B) 5 m
(C) 6 m (D) 7 m

Q.27 The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their curved surface area is
(A) 2 : 5 (B) 8 : 7
(C) 10 : 9 (D) 16 : 9

Q.28 The radii of two cylinder are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is
(A) 27 : 20 (B) 20 : 27
(C) 4 : 9 (D) 9 : 4

Q.29 The ratio between the radius of the base and the heights of a cylinder is 2 : 3. If its volume is 1617 cm^3, then its total surface area is
(A) 308 cm^2 (B) 462 cm^2
(C) 540 cm^2 (D) 770 cm^2

Q.30 Two circular cylinders of equal volume have their heights in the ratio 1 : 2. The ratio of their radii is
(A) $1 : \sqrt{2}$ (B) $\sqrt{2} : 1$
(C) 1 : 2 (D) 1 : 4

Q.31 The ratio between the curved surface area and the total surface area of a right circular cylinder is 1 : 2. If the total surface area is 616 cm^2, then the volume of the cylinder is
(A) 1078 cm^3 (B) 1232 cm^3
(C) 1848 cm^3 (D) 924 cm^3

Q.32 In a cylinder, if the radius is halved and the height is doubled, then the volume will be
(A) the same (B) doubled
(C) halved (D) four times

Q.33 The number of coins 1.5 cm in diameter and 0.2 cm thick to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm is
(A) 540 (B) 450
(C) 380 (D) 472

Q.34 The radius of a wire is decreased to one-third. If volume remains the same, the length will becomes
(A) 2 times (B) 3 times
(C) 6 times (D) 9 times

Q.35 The diameter of a roller, 1 m long, is 84 cm. If it takes 500 complete revolutions to level a playground, the area of the playground is
(A) 1440 m^2 (B) 1320 m^2
(C) 1260 m^2 (D) 1550 m^2

Q.36 2.2 dm^3 of lead is to be drawn into a cylindrial wire 0.50 cm in diameter. The length of the wire is
(A) 110 m (B) 112 m
(C) 98 m (D) 124 cm

Q.37 The lateral surface area of a cylinder is
(A) $\pi r^2 h$ (B) πrh
(C) $2\pi rh$ (D) $2\pi r^2$

Q.38 The height of a cone is 24 cm and the diameter of its base is 14 cm. The curved surface area of the cone is
(A) 528 cm^2 (B) 550 cm^2
(C) 616 cm^2 (D) 704 cm^2

Q.39 The volume of a right circular cone of height 12 cm and base radius 6 cm, is
(A) $(12\pi)cm^3$ (B) $(36\pi)cm^3$
(C) $(72\pi)cm^3$ (D) $(144\pi)cm^3$

Q.40 How much cloth 2.5 m wide will be required to make a conical tent having base radius 7 m and height 24 m ?
(A) 120 m (B) 180 m
(C) 220 m (D) 550 m

Q.41 The volume of a cone is 1570 cm^3 and its height is 15 cm. What is the radius of the cone ? (Use $\pi = 3.14$)
(A) 10 cm (B) 9 cm
(C) 12 cm (D) 8.5 cm

Q.42 The height of a cone is 21 cm and its slant height is 28 cm. The volume of the cone is
(A) 7356 cm^3 (B) 7546 cm^3
(C) 7506 cm^3 (D) 7564 cm^3

Q.43 The volume of a right circular cone of height 24 cm is 1232 cm^3. Its curved surface area is
(A) 1254 cm^2 (B) 704 cm^2
(C) 550 cm^2 (D) 462 cm^2

Q.44 If the volume of two cones be in the ratio 1 : 4 and the radii of their bases be in the ratio 4 : 5, then the ratio of their heights is
(A) 1 : 5 (B) 5 : 4
(C) 25 : 16 (D) 25 : 64

Q.45 If the height of a cone is doubled, then its volume is increased by
(A) 100% (B) 200%
(C) 300% (D) 400%

Q.46 The curved surface area of one cone is twice that of the other while the slant height of the latter is twice that of the former. The ratio of their radii is
(A) 2 : 1 (B) 4 : 1
(C) 8 : 1 (D) 1 : 1

Q.47 The ratio of the volumes of a right circular cylinder and a right circular cone of the same base and the same height will be
(A) 1 :3 (B) 3 : 1
(C) 4 : 3 (D) 3 : 4

Q.48 A right circular cylinder and a right circular cone have the same radius and the same volume. The ratio of the height of the cylinder to that of the cone is
(A) 3 : 5 (B) 2 : 5
(C) 3 : 1 (D) 1 : 3

Q.49 The radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. Then, their volumes are in the ratio
(A) 9 : 8 (B) 8 : 9
(C) 3 : 4 (D) 4 : 3

Q.50 If the height and the radius of a cone are doubled, the volume of the cone becomes
(A) 3 times (B) 4 times
(C) 6 times (D) 8 times

Q.51 A solid metalic cylinder of base radius 3 cm and height 5 cm is melted to make n solid cones of height 1 cm and base radius 1 mm. The value of n is
(A) 450 (B) 1350
(C) 4500 (D) 13500

Q.52 A conical tent is to accomodate 11 persons such that each person occupies 4 m^2 of space on the ground. They have 220 m^3 of air to breathe. The height of the cone is
(A) 14 m (C) 16 m (B) 15 m (D) 20 m

Q.53 The volume of sphere of radius 2r is
(A) $\frac{32\pi r^3}{3}$ (B) $\frac{16\pi r^3}{3}$
(C) $\frac{8\pi r^3}{3}$ (D) $\frac{64\pi r^3}{3}$

Q.54 The volume of a sphere of radius 10.5 cm is
(A) 9702 cm^3 (B) 4851 cm^3
(C) 19404 cm^3 (D) 14553 cm^3

Q.55 The surface area of a sphere of radius 21 cm is
(A) 2772 cm^2 (B) 1386 cm^2
(C) 4158 cm^2 (D) 5544 cm^2

Q.56 The surface area of a sphere is 1386 cm^2. Its volume is
(A) 1617 cm^3 (B) 3234 cm^3
(C) 4851 cm^3 (D) 9702 cm^3

Q.57 If the surface area of a sphere is (144π) m^2, then its volume is
(A) $(288\pi)m^3$ (B) $(188\pi)m^3$
(C) $(300\pi)m^3$ (D) $(316\pi)m^3$

Q.58 The volume of a sphere is 38808 cm^3. Its curved surface area is
(A) 5544 cm^2 (B) 8316 cm^2
(C) 4158 cm^2 (D) 1386 cm^2

Q.59 If the ratio of the volumes of two spheres is 1 : 8, then the ratio of their surface area is
(A) 1 :2 (B) 1 :4
(C) 1 : 8 (D) 1 : 16

Q.60 A solid metal ball of radius 8 cm is melted and cast into smaller balls, each of radius 2 cm. The number of such balls is
(A) 8 (B) 16
(C) 32 (D) 64

Q.61 A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is
(A) 4.2 cm (B) 2.1 cm
(C) 2.4 cm (D) 1.6 cm

Q.62 A solid lead ball of radius 6 cm is melted and then drawn into a wire of diameter 0.2 cm. The length of wire is
(A) 272 m (B) 288 m
(C) 292 m (D) 296 m

Q.63 A metallic sphere of radius 10.5 cm is melted and then recast into small cones, each of radius 3.5 cm and height 3 cm. The number of such cones will be
(A) 21 (B) 63
(C) 126 (D) 130

Q.64 How much lead shots, each 0.3 cm in diameter can be made from a cuboid of dimensions 9 cm × 11 cm × 12 cm ?
(A) 7200 (B) 8400 (D) 84000 (C) 72000

Q.65 The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is
(A) 12 m (B) 18 m
(C) 36 m (D) 66 m

Q.66 A sphere of diameter 12.6 cm is melted and cast into a right circular cone of height 25.2 cm. The radius of the base of the cone is
(A) 6.3 cm (B) 2.1 cm
(C) 6 cm (D) 4 cm

Q.67 A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of these balls are 1.5 cm and 12 cm. The radius of the third ball is
(A) 1 cm (B) 1.5 cm
(C) 2.5 cm (D) 0.5 cm

Q.68 The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratio of the surface areas of the balloons in two cases is
(A) 1 : 4 (B) 1 : 3
(C) 2 : 3 (D) 1 :2

Q.69 The volumes of the two spheres are in the ratio 64 : 27 and the sum of their radii is 7 cm. The difference of their total surface areas is
(A) 38 cm^2 (B) 58 cm^2
(C) 78 cm^2 (D) 88 cm^2

Q.70 A hemispherical bowl of radius 9 cm contains a liquid. This liquid is to be filled into cylinderical small bottles of diameter 3 cm and height 4 cm. How many bottles will be needed to empty the bowl ?
(A) 27 (B) 35
(C) 54 (D) 63

Q.71 A cone and a hemisphere have equal bases and equal volumes. The ratio of their heights is
(A) 1 :2 (B) 2 : 1
(C) 4 : 1 (D) $\sqrt{2}$: 1

Q.72 A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is :
(A) 1 : 2 : 3 (B) 2 : 1 : 3
(C) 2 : 3 : 1 (D) 3 : 2 : 1

Q.73 If the volume and the surface area of a sphere are numerically the same, then its radius is
(A) 1 unit (B) 2 units
(B) 3 units (D) 4 units

Q.74 Which is false in case of a hollow cylinder ?
(A) Curved surface area of a hollow cylinder = $2\pi h(R + r)$
(B) Total surface area of a hollow cylinder = $2\pi(R + r)(h + R - r)$
(C) Inner curved surface area of a hollow cylinder = $2\pi h(R - r)$
(D) Area of each end of a hollow cylinder = $\pi(R^2 - r^2)$

Q.75 Which is false ?
(A) Volume of a hollow sphere = $\frac{4}{3}\pi(R^3 - r^3)$
(B) Volume of a hemisphere = $\frac{2}{3}\pi R^3$
(C) Total surface area of a hemisphere = $3\pi R^2$
(D) Curved surface area of a hemisphere = πR^2

Q.76 For a right circular cylinder of base radius = 7 cm and height = 14 cm, which is false ?
(A) Curved surface area = 616 cm^2
(B) Total surface area = 924 cm^2
(C) Volume = 2156 cm^3
(D) Total area of the end faces = 154 cm^2

Q.77 Which is false ?
A metal pipe is 63 cm long. Its inner diameter is 4 cm and the outer diameter is 4.4 cm. Then,
(A) its inner curved surface area = 792 cm^2
(B) its outer curved surface area = 871.2 cm^2
(C) surface area of each end = 2.64 cm^2
(D) its total surface area = 1665.84 cm^2

ANSWER KEY

1.	C	**2.**	B	**3.**	B	**4.**	C
5.	A	**6.**	D	**7.**	B	**8.**	A
9.	C	**10.**	B	**11.**	B	**12.**	D
13.	C	**14.**	B	**15.**	D	**16.**	B
17.	D	**18.**	B	**19.**	D	**20.**	C
21.	D	**22.**	B	**23.**	B	**24.**	C
25.	B	**26.**	C	**27.**	C	**28.**	B
29.	D	**30.**	B	**31.**	A	**32.**	C
33.	B	**34.**	D	**35.**	B	**36.**	B
37.	C	**38.**	B	**39.**	D	**40.**	C
41.	A	**42.**	B	**43.**	C	**44.**	D
45.	A	**46.**	B	**47.**	B	**48.**	D
49.	A	**50.**	D	**51.**	D	**52.**	B
53.	A	**54.**	B	**55.**	D	**56.**	C
57.	A	**58.**	A	**59.**	B	**60.**	D
61.	B	**62.**	B	**63.**	C	**64.**	D
65.	C	**66.**	A	**67.**	C	**68.**	A
69.	D	**70.**	C	**71.**	B	**72.**	A
73.	C	**74.**	C	**75.**	D	**76.**	D
77.	D						